recent summary[9] of the various estimates shows a range from 16 to 50 MeV. A plot of the best-fit values of V_S versus $(N-Z)/A$ for our 11.7-MeV data is shown in Fig. 4. The solid line is the straight-line least-squares fit to the plotted points.

It is more realistic to apply a correction for the Coulomb potential before extracting the nuclear symmetry dependence of the potential. After careful consideration, the author of Ref. 8 took the Coulomb correction to be $0.4\,Z/A^{1/3}$ MeV. This correction was applied to the best-fit values of V_S for the 11.7-MeV data; the results are plotted in Fig. 5. These results, which were obtained from a generalized optical-model analysis, show definite evidence for a nuclear symmetry dependence of the real nuclear potential. The magnitude of V_1, as indicated by this work, is in reasonable agreement with the value obtained by the more extensive study reported by Perey.[8]

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In Ref. 8 a correlation was found between W_D and $(N-Z)/A$. For comparison the values of W_D that were obtained from the optical-model analysis of the 11.7-MeV data are plotted as a function of $(N-Z)/A$ in Fig. 6. It is likely that the value obtained for Ni^{58} is influenced by compound-nucleus contributions to the data. The solid line in Fig. 6 represents a straight line least-squares fit to the plotted points with the Ni^{58} point excluded. The slope thus obtained is in fair agreement with the values obtained by Perey from the analysis of 14.3-, 17-, and 22.2-MeV data. The value of W_S for a given value of $(N-Z)/A$ is lower by a few MeV than the values obtained from the analysis of Ref. 8 with the spherically symmetric optical model. It is reasonable to obtain a lower value of W_D in the present strong coupling analysis because one important absorption channel is treated explicitly.

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Parity Nonconservation in Nuclei*

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The influence of the known weak interactions on the parity impurity of nuclear states is discussed. Derivation of a parity nonconserving interaction rests on the assumption of a current-current hypothesis for the weak interactions. Consequently, observation of parity impurity effects would be an important confirmation of this hypothesis. A simple approximate method of treating the nuclear parity impurity is developed and applied in an effort to find experimental situations favorable to observation of effects due to such impurity. Parity-forbidden alpha decay from excited states of light nuclei and certain electromagnetic transitions in the heavy nuclei appear to be promising. Special attention has been paid to the internal conversion electrons from the 123-keV transition in Lu^{173} whose polarization is estimated to be about 0.4%. An effect on polarized neutrons analogous to "optical rotation" is also discussed.

I. INTRODUCTION

The motivation for examining parity nonconservation in nuclei is at least twofold. Firstly, it is desirable to test the parity conservation of all interactions[1] since the weak interactions, such as beta decay, are known not to conserve parity. This program has been largely fulfilled in that experiments[2–19] have already placed exceedingly small upper limits on parity nonconservation in either the electromagnetic or nuclear interactions. If experiment ultimately detects the small deviations to be discussed here, the known parity-

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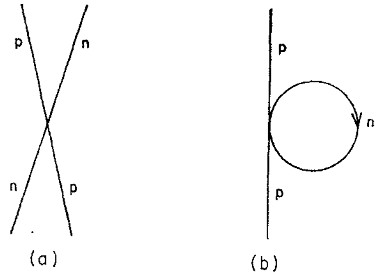

FIG. 1. Feynman diagrams for the lowest order weak interaction. The neutron (n) and proton (p) may be simultaneously interchanged.

violating weak interactions will presumably be responsible. Secondly, the strength of any parity nonconservation, henceforth assumed to arise by intervention of the weak interactions, gives some insight into the structure of these interactions. Several authors[20–22] have investigated the weak-interaction corrections to the pion-nucleon vertex.

The influence of the weak interactions on nuclear processes has also been examined theoretically by Krüger[23] and Blin-Stoyle.[24–26] This paper approaches the calculation of parity nonconserving effects in a somewhat different way from the above-named authors.

Conventionally,[27] the weak-interaction Lagrangian is written

$$\mathcal{L}_{\text{int}}=\sqrt{8}GJ_\mu J_\mu{}^\dagger. \qquad (1)$$

The theoretical and experimental work of recent years[28] has largely established the vector nature of the current, as anticipated in Eq. (1) above, and the contribution of both axial vector and polar vector currents together in J. The form of the current, suggested by considerations of symmetry and simplicity, is usually taken to be

$$J_\mu=(\bar{e}\gamma_\mu a\nu_1)+(\bar{\mu}\gamma_\mu a\nu_2)+(\bar{n}\gamma_\mu ap)+\cdots, \qquad (2)$$

where $a=\frac{1}{2}(1+i\gamma_5)$.

Recent experimental evidence[29] indicates a distinction between the neutrino associated with the electron in the weak interactions and the neutrino associated with the muon: This has been indicated in Eq. (2) by the subscripts 1 and 2 referring respectively to the electron- and muon-neutrino. Equations (1) and (2) together predict processes not yet observed, such as neutrino-electron scattering, and verification of such processes would constitute an important confirmation of the theory.

Observation of self-interaction terms in the postulated beta-decay interaction, as for example $(\bar{n}p)(\bar{n}p)^+$, presents just such a situation; the initial and final particles are the same. In this paper, the $(\bar{n}p)(\bar{n}p)^+$ interaction is discussed extensively, with only a comment on the lepton "self" terms below.

A. The Lepton "Self" Terms

Although the coupling $(\bar{e}\nu_1)(\bar{e}\nu_1)^+$ could be tested directly from the elastic scattering reaction $\nu_1+e\to\nu_1+e$, the experiment is greatly complicated by the small cross section[27] expected from the theory. The cross section increases with the center-of-momentum energy, but this feature cannot be utilized, as presently there are no intense high-energy beams of ν_1 (or $\bar{\nu}_1$). Even if such beams were available, the c.m. energy is much less for electrons than for nucleons resulting in a large relative cross section for obscuring processes such as $\nu_1+p\to n+\bar{e}$ or $\bar{\nu}_1+n\to p+e$. Reines[30] has proposed an experiment utilizing the high-intensity, low-energy antineutrinos from an atomic pile.

Astrophysical[31] significance has been attributed to the $(\bar{e}\nu_1)(\bar{e}\nu_1)^+$ coupling since stellar energy loss via neutrino pair production would become important at high temperatures.[32,33]

Other experiments are generally complicated by competition from electromagnetic processes: The $^3S_1$ state of positronium is predicted from Eqs. (1) and (2) to decay into $\nu+\bar{\nu}$, but the rate for this decay is only

$$3G^2m^5\alpha^3/16\pi^2=5\times10^{-11}\ \text{sec}^{-1},$$

giving a branching fraction

$$\tau(^3S_1\to3\gamma)/\tau(^3S_1\to\nu_1+\bar{\nu}_1)=7\times10^{-18}, \qquad (3)$$

while the $^1S_0$ state is stable to this decay mode due to helicity requirements. The branching fraction is quite sensitive to the mass of the particle, but even for the muon-antimuon system, Eq. (3) gives only 1.3×10^{-8}. In this latter system the natural muon decay itself competes unfavorably:

$$\frac{\frac{1}{2}\tau(^3S_1\to e+\bar{\nu}_1+\nu_2+\bar{\mu})}{\tau(^3S_1\to\nu_2+\bar{\nu}_2)}=18\pi\alpha^3=2.2\times10^{-5}.$$

B. The Intermediate Vector Boson Hypothesis

It is possible that the weak interactions are mediated by a vector boson, and if this is correct, then $J_\mu J_\mu{}^+$ in Eq. (1) should be replaced by $J_\mu D_{\mu\nu}J_\nu{}^+$, where

$$D_{\mu\nu}=(\delta_{\mu\nu}M_x{}^2-q_\mu q_\nu)/(M_x{}^2-q^2), \qquad (4)$$

with M_x=boson mass and q_μ=four-momentum trans-

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fer. The effect on the range of interaction is the only feature of the boson considered in this paper.

II. THE INTERACTION

A. Lowest Order

The Feynman diagrams for the lowest-order weak interactions involving just nucleons are given in Fig. 1.

The interaction, ignoring for the moment vector boson contributions and pionic corrections, is in lowest-order of perturbation theory

$$H_{\text{int}}=\sqrt{8}G(\bar{n}_1\gamma_\mu a p_1)(\bar{p}_2\gamma_\mu a n_2)\,, \tag{5}$$

where the notation of Ref. 27 is used throughout, specifically $a=\frac{1}{2}(1+i\gamma_5)$, $GM^2=1.01\pm0.01\times10^{-5}$, M is the proton mass,

$$\boldsymbol{\gamma}=\beta\boldsymbol{\alpha}\,,\quad \gamma_t=\beta\,,\quad \gamma_5=\gamma_x\gamma_y\gamma_z\gamma_t\,,\text{ etc.}$$

The self-energy diagram in Fig. 1(b) might be of interest since it could introduce pseudoscalar and pseudovector terms into the nucleon propagator and thereby be ultimately a source of parity nonconserving effects. No pseudoscalar term can be generated if the fundamental weak interaction couplings are γ_μ and $i\gamma_\mu\gamma_5$, since such a pseudoscalar would give time-reversal noninvariant effects while the interaction described will not. The pseudovector contribution must result in a nucleon propagator of the form $(\not{p}+i\eta\gamma_5\times\not{p}-M)^{-1}$, where $(\not{p}=\gamma_\mu p_\mu)$, or $(\not{p}+i\eta\gamma_5\not{p}-M)\psi_N=0$ for the free nucleon. If all nucleon states are transformed by $\psi_N'=\exp[-i\gamma_5\sinh^{-1}(\eta/2)]\psi_N$, the new states then obey $(\not{p}-M)\psi_N'=0$ and the correction to the propagator may be removed by a gauge transformation. The consequence of this is to change the electromagnetic coupling $(\bar{\psi}\gamma_\mu\psi)$ to $(\bar{\psi}\{\gamma_\mu-i\eta\gamma_5\gamma_\mu\}\psi)$ which would destroy gauge invariance; hence the corrections to electromagnetism given by diagrams such as Fig. 1(b) with a photon coupled to the vertex must give $\gamma_\mu\rightarrow\gamma_\mu+i\eta\gamma_5\gamma_\mu$ and altogether $(\bar{\psi}\gamma_\mu\psi)\rightarrow(\bar{\psi}'\gamma_\mu\psi')$. A pseudovector meson coupling is transformed $\boldsymbol{q}\gamma_5\rightarrow\boldsymbol{q}\gamma_5+i\eta\boldsymbol{q}$ and the extra term gives no contribution, while the pseudoscalar coupling is unchanged, hence it leads to no effects here.

Equation (5) may also be written

$$\sqrt{8}G[(a\bar{N}_1)\gamma_\mu(aN_1)]T_{12}[(a\bar{N}_2)\gamma_\mu(aN_2)]\,,$$

where N is the combined isotopic spinor (1×2) and Dirac spinor (1×4). The proton isotopic spinor is $\begin{pmatrix}1\\0\end{pmatrix}$ and the neutron isotopic spinor is $\begin{pmatrix}0\\1\end{pmatrix}$. T_{12} is the isotopic spin operator necessary to reproduce (6) and is given by $(\tau_+{}^1\tau_-{}^2+\tau_-{}^1\tau_+{}^2)$, where $\tau_+\begin{pmatrix}1\\0\end{pmatrix}=0$; $\tau_-\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$; $\tau_-\begin{pmatrix}0\\1\end{pmatrix}=0$; and $\tau_+\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$. The interaction can be thought of simply as a polar vector interaction between the (aN) components (spin antiparallel to the momentum vector) of the two nucleons. Thus, the nucleons are "reminded" of their intrinsic handedness when involved in the weak interactions, and have a tendency to correlate their spins with the momentum vector, as would be occasioned by terms of the form $\boldsymbol{\sigma}\cdot\mathbf{p}$ in the Hamiltonian. Parity is no longer a good quantum number if the Hamiltonian contains such pseudoscalars. The parity admixing can be seen another way by considering a classical system of a nucleon orbiting a force center. If the spin and orbital angular momentum vectors are initially parallel, then a $\boldsymbol{\sigma}\cdot\mathbf{p}$ interaction will produce a torque causing the spin vector to precess out of alignment with the orbital angular momentum vector. To conserve total angular momentum, the orbital angular momentum must increase. The quantum-mechanical analog would be, for example, a $P_{3/2}$ state which must admix $D_{3/2}$ states to be stationary. The actual nonrelativistic form of Eq. (5) is slightly more complicated than $\boldsymbol{\sigma}\cdot\mathbf{p}$ and may be written

$$\begin{aligned}&-8^{1/2}G\frac{1}{8M}(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)\cdot\{(\mathbf{p}_2-\mathbf{p}_1),\delta(1,2)\}_+\\&-8^{1/2}G\frac{i}{8M}(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_1)\cdot[(\mathbf{p}_2-\mathbf{p}_1),\delta(1,2)]_-.\end{aligned}\tag{6}$$

The curly brackets followed by the subscript $(+)$ indicates the anticommutator, i.e., $\{A,B\}_+=AB+BA$. The commutator is denoted by square brackets followed by the subscript $(-)$.

B. The General Form

The most general *polar vector* current is

$$J_\mu{}^V(q^2)=C_{1V}(q^2)\gamma_\mu+C_{2V}(q^2)i\sigma_{\mu\nu}q_\nu+C_{3V}(q^2)iq_\mu\,, \tag{7}$$

where $\sigma_{\mu\nu}=\frac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$ and q_μ is the four-momentum transfer with $q^2=q_\mu q_\mu$. If the CVC theory[27,34] obtains, the additional condition $q_\mu J_\mu{}^V=0$ must be satisfied which requires $C_{3V}=0$. Furthermore, we may identify $C_{1V}(q^2)=F_{1V}(q^2)$ and $C_{2V}(q^2)=F_{2V}(q^2)$, where F_{1V} and F_{2V} are the nuclear isovector charge and magnetic moment form factors as defined by Hofstadter.[35] The subscript V appearing on G and F refers to vector properties in different spaces, Lorentz for G and isotopic spin for F. $C_{3V}(q^2)$ is expected to be zero even if the CVC theory is not valid: If γ_μ is regarded as the fundamental coupling, then strong interaction corrections cannot induce G_{3V} without violating known strong interaction symmetries.[36] Under charge conjugation γ_μ and $i\sigma_{\mu\nu}q_\nu$ transform alike while iq_μ transforms differently, and consequently, iq_μ cannot be produced by radiative corrections of interactions invariant under

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charge conjugation, as are the strong[37] and electromagnetic interactions. This result is familiar for the electromagnetic interaction. Several simplifications follow from the CVC theory. For one, the additional momentum-transfer terms in the boson propagator, Eq. (4), do not contribute in lowest order, and we may treat the boson here as a scalar meson.

The most general axial vector current is

$$J_\mu{}^A(q^2)=[C_{1A}(q^2)\gamma_\mu+C_{2A}(q^2)\sigma_{\mu\nu}q_\nu+C_{3A}(q^2)q_\mu]i\gamma_5. \quad (8)$$

The assumption that $i\gamma_\mu\gamma_5$ is the fundamental coupling generating the axial vector current eliminates the terms that transform differently from $i\gamma_\mu\gamma_5$ under charge[37] conjugation. This requires that $C_{2A}(q^2)=0$. Furthermore, the $iq_\mu\gamma_5$ term fails to contribute for a vanishing divergence of $J_\mu{}^V$. This "induced pseudoscalar" term will contribute to the tiny parity *conserving* part of the weak interaction, negligible compared to strong parity conserving interactions. All that is known experimentally about $C_{1A}(q^2)$, equal to $-G_A/G_V$ in the usual notation,[38] is the magnitude near zero-momentum transfer

$$C_{1A}(0)\equiv\lambda=1.20\pm0.04.$$

C. The Interaction (CVC)

The CVC theory offers an alternative method of introducing the parity nonconserving interaction. Here, the axial vector current is treated on the same footing as the electromagnetic field, and is introduced into the Hamiltonian in the same way, namely,

$$p_{1\mu}\to p_{1\mu}-eA_\mu(\mathbf{r}_1)\tfrac{1}{2}(1+\tau_2{}^1) \quad (9)$$

becomes

$$p_{1\mu}\to p_{1\mu}-\tfrac{1}{4}\sqrt{8}G[\bar{u}(\mathbf{r}_1)i\gamma_\mu\gamma_5u(\mathbf{r}_1)]_2 \times(\tau_+{}^1\tau_-{}^2+\tau_-{}^1\tau_+{}^2). \quad (10)$$

Here, the subscript 2 refers to the particle generating the field seen at the position of particle 1, and e is the proton charge. For the moment, the vector boson and form factor considerations have been set aside to simplify the expressions. The nonrelativistic reductions for the axial vector current are

$$[\bar{u}(\mathbf{r}_1)i\boldsymbol{\gamma}\gamma_5u(\mathbf{r}_1)]_2=-\boldsymbol{\sigma}_2\delta^3(\mathbf{r}_1-\mathbf{r}_2)\,, \quad (11)$$

$$[\bar{u}(\mathbf{r}_1)i\gamma_0\gamma_5u(\mathbf{r}_1)]_2=-(1/2M)\{\boldsymbol{\sigma}_2\cdot\mathbf{p}_2,\delta^3(\mathbf{r}_1-\mathbf{r}_2)\}_+, \quad (12)$$

where the equivalent *operator* has been written on the right-hand side, and the delta function derives from the implicit assumption that particle 2 acts only at the position of particle 1.

The nonrelativistic Hamiltonian for a charged spin-$\frac{1}{2}$ particle interacting with the electromagnetic field is

$$\frac{1}{2M}[\mathbf{p}-e\mathbf{A}(\mathbf{r})]^2+e\varphi(\mathbf{r})-(\mu+1)\frac{e}{2M}(\boldsymbol{\sigma}\cdot\mathbf{B})+\cdots, \quad (13)$$

where μ is the anomalous magnetic moment. For two particles, each acting as an axial-vector current source of interaction for the other, Eq. (13) together with the substitutions (10) through (12) give

$$-\left[\frac{8^{1/2}G}{8M}(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)\cdot\{(\mathbf{p}_2-\mathbf{p}_1),\delta(1,2)\}_+ +(\mu^v+1)\frac{8^{1/2}G}{8M}(i\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_1)\cdot[(\mathbf{p}_2-\mathbf{p}_1),\delta(1,2)]_-\right]T_{12}, \quad (14)$$

which is just Eq. (6) with an additional correction for the anomalous moment. The contribution of Eq. (14) to the electromagnetic interaction is given by applying substitution (9) which yields for nucleons

$$\frac{e8^{1/2}G}{8M}(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)\cdot\mathbf{A}(\mathbf{r}_1)\delta(1,2)i(\boldsymbol{\tau}^1\times\boldsymbol{\tau}^2)_z. \quad (14')$$

In principle, the contribution by any nuclear potential to the parity nonconserving forces via substitution (10) can be computed. For electromagnetism, it has not been possible to compute such corrections from the nuclear interactions mainly due to ignorance of the structure of the strong interactions. The corrections are no easier to compute for the parity nonconserving interaction than for electromagnetism, and the interaction (14) is therefore necessarily incomplete. There does not appear to be any advantage then to including such parity nonconserving forces as are *generated* by the spin-orbit forces in Eq. (13), since even more important terms of this form are known to be generated by the nuclear forces. Although the nuclear forces can be treated phenomenologically, it does not follow that corrections computed from such a treatment actually constitute any improvement. For example, when the exchange of charged pions, etc., among nucleons is replaced by an effective static potential, the substitution (9) is incomplete as the current carried by these charges is concealed. These problems are familiar for electromagnetism, but they differ in scope for the parity nonconserving interactions due to the vast difference in the range of interaction. Many electromagnetic features, such as the Coulomb energy of a nucleus, are relatively insensitive to the detailed distribution of charge and currents near the individual nucleons. The parity nonconserving forces act, however, only when two nucleons are close together, and analogy to electromagnetism cannot be applied to argue away effects of the detailed nuclear interactions. It might be argued to the contrary that, since the nuclear interaction seems to exhibit a

[37] Charge conjugation is usually combined with a rotation of 180° about the "y" axis in isotopic spin space and called G conjugation. The strong interactions are invariant under the latter transformation; however, the essential difference between the interaction currents iq_μ and γ_μ comes from their charge conjugation properties.

[38] O. M. Kofoed-Hansen and C. J. Christensen, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1962), Vol. 41, Part 2, p. 88.

strong repulsive core, it is rare for two nucleons to be sufficiently close together to feel the weak interactions. This point will be examined in the next section and the difficult but important question of additional contributions to Eq. (14) set aside.

D. Effect of a Repulsive Core

In the center-of-mass system of two nucleons, $q^2=-Q^2$, where **Q** is the three-momentum transfer, our collection of form factors and the boson propagator given in Eq. (4) produces in Eq. (5) such factors as

$$C_{1V}(-Q^2)C_{1A}(-Q^2)(M_x^2/M_x^2+Q^2),$$

whose Fourier transform is the "potential." The form of C_{iV} is deduced from experimental measurements on the electromagnetic form factors, F_{iV}, of the nucleon by using the CVC theory as discussed in Sec. IIB. The experimental data of F_{iV} are summarized[35] as

$$F_{iV}(-Q^2)=(1-v_i)+(v_iM_v^2/M_v^2+Q^2),$$

where i equals 1 or 2, $M_v=600$ MeV, $v_1=0.92\pm0.10$, and $v_2=1.10$. If the momentum dependence of C_{1A} is ignored along with the small difference between $i=1$ and 2 ($v_i\approx1$), the potential becomes

$$f(r)=\lambda\frac{M_v^2M_x^2}{(M_v^2-M_x^2)}\frac{1}{4\pi r}(e^{-M_xr}-e^{-M_vr}). \quad (15)$$

Practical calculations are generally made with nuclear wave functions having no interparticle correlations due to a repulsive hard core. For such calculations, there is little point in using Eq. (15) in preference to $\lambda\delta(r)$. On the other hand, $f(r)$ for r less than r_c, the radius of the core, will not contribute if wave functions containing the hard-core correlation are used. Thus, an estimate of the effect of the core is simply to use the uncorrelated wave functions along with $\lambda'\delta(r)$ where

$$\lambda'=\lambda\int_{r_c}^{\infty}f(r)4\pi r^2dr.$$

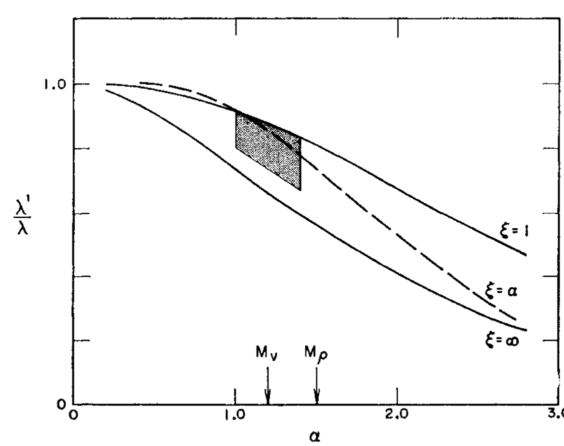

FIG. 2. Plot of λ'/λ as a function of α and ξ.

Writing $M_vr_c=\alpha$, $M_x/M_v=\xi$, we find

$$\lambda'=\lambda(\xi^2-1)^{-1}[\xi^2(1+\alpha)e^{-\alpha}-(1+\xi\alpha)e^{-\xi\alpha}],$$

which is plotted in Fig. 2. The shaded portion corresponds to $r_c=0.40\pm0.05F$, $M_v^2=(18\pm2)M_\pi^2$, and an arbitrary estimate $\xi=2\pm1$. Figure 2 suggests $\lambda'\sim0.8\lambda\sim1.0$ and therefore λ' has been set equal to unity for numerical computations throughout this paper. Replacement of λ by λ' is presumed to correct for repulsive core effects.

III. ANALYSIS

A. General Considerations

Angular Momentum

Interaction (14) transforms as a pseudoscalar under combined rotation of the space and spin coordinates; thus, the total angular momentum is conserved, but not necessarily the spin or orbital angular momenta separately. Conservation of total angular momentum is built into the interaction by the choice of possible couplings. At least one group[6] has interpreted a parity impurity experiment alternatively in terms of an upper limit on the angular momentum impurity of a nuclear level. There is, however, at present, no experimental or theoretical association between the weak interactions and any possible angular momentum violating interaction. Since all scalar and pseudoscalar interactions automatically conserve J and m_J, an interaction violating these quantum numbers would necessarily consist of an incomplete tensor such as the z component of a vector or the zz component of a vector product. Such terms appear when the Hamiltonian is incompletely formulated as, for example, by including the magnetic interaction between the nuclear spin and the atomic electrons, but not otherwise including the electrons in the Hamiltonian.

Isotopic Spin

The isotopic spin dependence of Eq. (14) can be written

$$T_{12}=(\tau_+{}^1\tau_-{}^2+\tau_-{}^1\tau_+{}^2)$$
$$=\tfrac{2}{3}T_{12}{}^{(0)}+T_{12}{}^{(2)}, \quad (16)$$

where

$$T_{12}{}^{(0)}=\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2,$$
$$T_{12}{}^{(1)}=\boldsymbol{\tau}^1\times\boldsymbol{\tau}^2,$$
$$T_{12}{}^{(2)}=\tfrac{1}{3}\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2-\tau_z{}^1\tau_z{}^2.$$

No $T_{12}{}^{(1)}$ can appear since the Pauli principle requires the interaction to be symmetric under combined exchange of spin, space, and isotopic spin. The corrections from the strong interactions, as illustrated in Fig. **3**, may alter Eq. (16) to

$$T_{12}=\tfrac{1}{3}(2+\epsilon_1-\epsilon_2)T_{12}{}^{(0)}+(1-\epsilon_1+\epsilon_2)T_{12}{}^{(2)}+\epsilon_1+\epsilon_2, \quad (17)$$

where Fig. 3(d), for example, illustrates the effective coupling $(\bar{p}p)(\bar{p}p)^+$ via the isotopic spin dependence

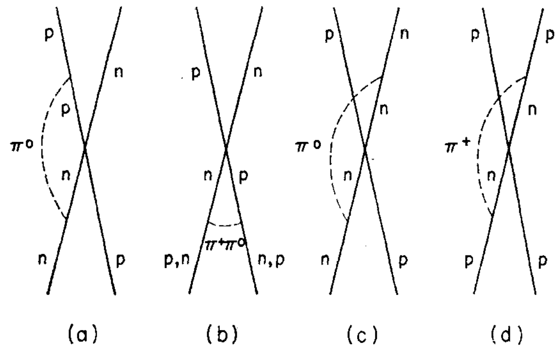

FIG. 3. Strong interaction corrections to the $(\bar{n}p)(\bar{n}p)^+$ interaction.

$\frac{1}{2}(1+\tau_z{}^1\tau_z{}^2)$, while Fig. 3(b) gives $(\bar{n}p)(\bar{p}n)^+$ with dependence $\frac{1}{2}(1-\tau_z{}^1\tau_z{}^2)$. The two additional couplings have been arbitrarily assigned the relative amplitudes $2\epsilon_1$ and $2\epsilon_2$ in Eq. (17). Strictly $\epsilon_1(\epsilon_2)$ should appear as the coefficient on a force with slightly shorter range; however, the difficulty in actually performing the required calculations is well known, and $\epsilon_1(\epsilon_2)$ in Eq. (17) should be regarded merely as a crude imitation of corrections such as illustrated in Fig. 3.

The appearance of $T_{12}{}^{(2)}$ in Eq. (16) is significant in that the usual isotopic spin selection rules may be circumvented. First-order electromagnetic and beta-decay transitions are forbidden between states with $\Delta T=2$. The operator $T_{12}{}^{(2)}$ admixes parity impure states differing from the unperturbed state by two units of isotopic spin. Thus, for $\Delta T=2$, the regular (parity allowed) electromagnetic and beta-decay transitions are forbidden, while the irregular (parity forbidden) transitions are allowed. For example, a hypothetical beta-decay transition between a $(J^\pi,T)=(0^-,2)$ initial state and a $(0^+,0)$ final state must proceed by a T-forbidden first-forbidden decay, but is completely allowed from an admixed amplitude of $(0^+,0)$ in the initial state or $(0^-,2)$ in the final state.

Experimental detection of a $\Delta T=2$ beta- or gamma-ray transition would not alone support interaction (14), since the Coulomb forces also lead to isotopic spin impurity of the levels. A state identified with a certain value of T will, in general, have, due to Eq. (14), parity impure admixtures of T, $T\pm1$, and $T\pm2$. Only for $T\geq 2$ does the full range become possible; otherwise, $T=0\rightarrow T=0$, 2 and $T=1\rightarrow T=1$, 2, 3.

Time-Reversal Invariance

The interaction (1) assumed is invariant under time reversal. The validity of this assumption has not been tested to great accuracy, although the apparent complete violation of both parity and charge conjugation invariance, together with the *TCP* theorem, indicate the weak interactions are invariant under time reversal. It follows from time-reversal invariance[39] that: (a) A quantity odd under the parity transformation has zero *expectation* value despite the presence of the parity nonconserving interaction, in contrast to: (b) A quantity even under the parity transformation has its expectation value unaltered (to order G^2) by effects from the parity nonconserving interaction. Assertion (b) follows directly from the parity selection rule. The parity of a nuclear state is therefore unchanged (to order G^2) as a trivial consequence of (b), and it is unnecessary to develop a special notation, such as complex or fraction parity, to describe parity impure states.

The odd-parity static moments (electric dipole, magnetic quadripole, electric octopole, etc.) are required by (a) to vanish despite the presence of interaction (14), and experiments to detect these moments do not test (14).

Assertion (a) can be phrased equivalently as requiring the phase of the regular and irregular wave functions to be relative imaginary. A useful application is the effect of any time-reversal invariant parity nonconserving force on a single-particle wave function, in which case the radial dependence of the number of nodes n and orbital angular momentum l may be separated from the spin and angular coordinates coupling the spin $s=\frac{1}{2}$ and l into $j=l\pm\frac{1}{2}$ to give $\psi=R_{nl}(r)|sljm\rangle$. The only wave functions that can be admixed are $R_{n'l'}(r)|sl'jm\rangle$, where $l'=2j-l$. However, the operator $\boldsymbol{\sigma}\cdot\hat{r}(\hat{r}=\mathbf{r}/r)$ has the property

$$-(\boldsymbol{\sigma}\cdot\hat{r})|sljm\rangle=|sl'jm\rangle \quad (l'=2j-l)\,, \tag{18}$$

and the irregular wave function can then be written $iG\boldsymbol{\sigma}\cdot\hat{r}0(r)\psi$, with $0(r)$ a scalar operator acting only on the radial wave function. To order G^2, the new wave functions are given by

$$\tilde{\psi}=\exp[iG\boldsymbol{\sigma}\cdot\hat{r}0(r)]\psi\,. \tag{19}$$

It is useful to note that the couplings dropped from Eqs. (7) and (8) generate parity nonconserving interference terms that violate time-reversal invariance. Similarly, a scalar interaction $J^S=C_{1S}$ and a pseudoscalar interaction $J^P=C_{1P}\gamma_5+C_{2P}\mathbf{q}\gamma_5$ can only give parity nonconservation together with violation of time-reversal invariance. The scalar coupling $J^{S'}=C_{2S}i\mathbf{q}$ essentially vanishes for nucleons but otherwise would give parity nonconservation plus time-reversal invariance and, in fact, the interference term with J^P has the same structure as would be given by the $-q_\mu q_\nu/M_x{}^2(M_x{}^2-q^2)$ part of the propagator (4). The tensor parity nonconserving interaction $(\sigma_{\mu\nu})_1(\sigma_{\mu\nu}\gamma_5)_2$ violates time-reversal invariance, and therefore expression (14), after modifications of the form (15) and (17), represents the most general parity nonconserving, time-reversal invariant, Hermitian, two-nucleon, pseudoscalar potential.

B. Single-Particle Approximation

With an expression as complicated as (14), it is difficult to make any estimates without having rather

[39] E. P. Wigner, *Group Theory* (Academic Press Inc., New York, 1959), p. 344.

detailed wave functions for each specific problem. A natural approximation in a many-body system is to select a specific particle and average the interaction over the remaining particles. This has been done in Appendix A with the result that Eq. (14) reduces to the single-particle operator

$$H_{\text{int}}\approx 2G'\boldsymbol{\sigma}\cdot\mathbf{p}\tfrac{1}{2}[1+(N-Z)\tau_z/A]=G''\boldsymbol{\sigma}\cdot\mathbf{p}\,, \quad \text{(A6)}$$

where

$$G'=8^{1/2}G\lambda'(\mu^v+1)\rho_0/8M\,.$$

$$\rho_0=\text{nucleon density in the nucleus}\,,$$

and $\mathbf{p}$ now refers to a fixed coordinate system at the center of mass of the nucleons (other than 1). The averaged isotopic spin part requires a proton ($\langle\tau_z\rangle=+1$) to interact only with $\frac{1}{2}[1+(N-Z)/A]$ or N/A of the density, i.e., just with the neutrons and vice versa. Note that the factor $(\mu^v+1)=4.70$ arises from the CVC theory, and would necessarily be estimated ≈ 1 in the absence of this theory.

A simple model Hamiltonian with the above (A6) interaction such as

$$H=p^2/2M+V(r)+G''\boldsymbol{\sigma}\cdot\mathbf{p}=H_0+G''\boldsymbol{\sigma}\cdot\mathbf{p} \quad (20)$$

can be solved to order G^2 by the substitution [compare with (10) and (11)]

$$\mathbf{p}\rightarrow\mathbf{p}-MG''\boldsymbol{\sigma}$$

or equivalently [compare Eq. (19)]

$$H=e^{iS}H_0e^{-iS}\,;\quad \psi_k=e^{iS}\psi_k^{(0)}\,, \quad (21)$$

where $S=MG''\boldsymbol{\sigma}\cdot\mathbf{r}$ and $\psi_k^{(0)}$ are the eigenstates of H_0. Such a transformation is especially useful since the matrix elements of an operator A between the impure parity states of Eq. (20) are equal to the matrix elements of

$$e^{-iS}Ae^{iS}=A-i[S,A]+0(G^2)=A+\tilde{A} \quad (22)$$

between the unperturbed states. It is often simpler in actual calculation to use $-i(SA-AS)$ and the properties of the operator $\boldsymbol{\sigma}\cdot\hat{r}$ [Eq. (18)] rather than using the operator given by explicitly computing $-i[S,A]$. The operator $-i[S,A]$ is denoted by $\tilde{A}$ in this paper: the operator $\tilde{M}1$, for example, would be $-i[S,M1]$ and has selection rules similar to an $E1$ transition. If A itself has an irregular part generated by the interaction (14), then $\tilde{A}$ is understood to be $-i[S,A]+A_{\text{irreg}}$.

Historically,[40] the degree of admixing is described by $\mathfrak{F}$ ($\tilde{\psi}=\psi+i\mathfrak{F}\psi'$) which may now be conveniently defined as $\mathfrak{F}=MG''R$, where $R=R_0A^{1/3}$, ($R_0=1.2\times10^{-13}$ cm), giving typical numerical results ($MG''R_0=1.2\times10^{-7}$ for $N=Z$).

$$\begin{aligned} A&=16; & \mathfrak{F}&=3.1\times10^{-7}\\ A&=160; & \mathfrak{F}&=8.0\times10^{-7}\text{ for an extra proton}\\ && &=5.5\times10^{-7}\text{ for an extra neutron}. \end{aligned} \quad (23)$$

[40] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

A quantity $\mathfrak{R}$ is usually defined such that

$$i\mathfrak{R}\mathfrak{F}=-\frac{(\|\text{irregular transition}\|)}{(\|\text{regular transition}\|)}\,, \quad (24)$$

with $\mathfrak{R}$ containing the explicit details after factoring out the strength, $\mathfrak{F}$, of the parity nonconserving interaction. In practice, $\mathfrak{R}$ has been estimated by comparing "typical" rates for both types of transition; however, we may use Eqs. (24) and (21) together with $\mathfrak{F}=MG''R$ to estimate

$$\mathfrak{R}=\frac{(\|[\boldsymbol{\sigma}\cdot\mathbf{r}/R,A_{\text{irreg}}]\|)}{(\|A_{\text{regular}}\|)}\,. \quad (25)$$

In Eq. (24) the matrix elements are taken between the perturbed states, while in Eq. (25) the unperturbed wave functions are to be used.

Several interesting deductions can be made in this simple model which should remain as *approximate* features of any results using Eq. (14) and a more sophisticated nuclear Hamiltonian.

(1) Electric multipole transitions will *not* be admixed into the corresponding magnetic multipole. To simplify the description, we denote $EL(ML)$ as representing an EL transition in which the parity impurities relax the selection rules to allow an admixture of ML: we are considering here the $ML(EL)$ transitions. The EL transition operator, ignoring the tiny magnetic contributions, is proportional to the corresponding static moment operator and therefore commutes with S in Eq. (22), hence $\tilde{E}L=-i[S,EL]=0$. See Sec. IVD.

(2) $EL(ML)$ transitions will be admixing, since $-i[S,ML]\neq 0$ as is discussed in more detail in Sec. IVD.

(3) The admixing is independent of $V(r)$, consequently admixing should be independent of the level structure. One might suppose that two levels of opposite parity, but the same spin, would admix more strongly if close together as suggested by the energy denominator in the perturbation theory expression for admixing.

$$\tilde{\psi}_k=\psi_k+\sum_{n\neq k}\frac{\langle\text{Expression (14)}\rangle_{nk}}{E_k-E_n}\psi_n+\cdots.$$

However, this argument fails to the extent that the perturbation can be removed by a gauge transformation. Furthermore, no requirement of spherical symmetry has been imposed, and the arguments apply unchanged to particles bound in a deformed potential such as used in the Nilsson[41] model.

(4) No $\Delta T=2$ effects appear within the single-particle approximation as can be seen from the form of Eq. (A6).

It would be an interesting extension of our results if the spin-orbit force could be included in Eq. (20);

[41] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

however, S and $\boldsymbol{\sigma}\cdot\mathbf{l}$ do not commute. It is generally assumed that the spin-orbit contribution is from regions near the nuclear surface, although the origin of the force is poorly understood, and we have already thrown away surface terms when approximating the nuclear density as being everywhere constant. Furthermore, any spin-orbit force should induce its own parity nonconserving interaction via Eq. (10) and it is not correct to simply add $u(r)\boldsymbol{\sigma}\cdot\mathbf{l}$ to Eq. (20). Thus, the approximation $\bar{\psi}=e^{iS}\psi$ is probably as good with the spin-orbit force as without. However, in Sec. IVD, it will be assumed, for the purpose of obtaining order-of-magnitude estimates of $ML(EL)$ transitions, that a force $(a/2)\boldsymbol{\sigma}\cdot\mathbf{l}$ is simply added to the nuclear Hamiltonian in Eq. (20).

IV. EXPERIMENTAL PREDICTIONS

A. General Effects

Most of the comments in this subsection have been made elsewhere[1] and are repeated only for the sake of coherence. An easily understood phenomenon that is due to admixing of irregular parity states is the polarization of decay products. If an $E1$ gamma-ray transition, e.g., $1^-\rightarrow 0^+$ contains traces of $1^+\rightarrow 0^+$ and $1^-\rightarrow 0^-$, then $M1$ is admixed. Classically, this would correspond to an oscillating electric dipole with a superimposed oscillating magnetic dipole 90° out of phase. At a certain instant the electric field seen by a distant observer will be entirely due to the electric dipole and parallel to the dipole direction. A quarter of a period later, the electric field is now due to the magnetic dipole and the field is perpendicular to the dipole direction. Thus, an elliptically polarized outgoing wave rather than a plane polarized wave should be observed. In other words, the tendency of a nucleon to align its spin with its direction of motion can be transferred to the outgoing radiation. An example of such an effect is the circular polarization of inner bremsstrahlung from polarized electrons in beta decay. Similarly, the scattering of initially unpolarized particles can result in a polarization along the momentum vector of the outgoing particles.

If a decaying nucleus is polarized in a definite direction and emits polarized radiation, that radiation will tend to be emitted parallel (or antiparallel, depending on the specific details) to the direction of polarization of the decaying nucleus. Thus an alternative to measuring the helicity of weakly polarized particles is to measure their angular distribution, which contains odd powers of $\cos\theta$, from polarized nuclei. If parity nonconserving interactions are used to both polarize the decaying nucleus *and* outgoing radiation, the odd powers of $\cos\theta$ correlation are of order $\mathfrak{F}^2$. Thus, investigators[11,12] have preferred to excite and polarize the nuclei via beta decay (beta-gamma correlation) or polarized neutron capture (n-gamma correlation). In principle, an inverse method such as gamma-beta correlation could be used, but the long beta-decay lifetimes (shortest known N^{12} is 0.01 sec) permit depolarization effects to wash out all angular correlation.

Formulas for the magnitude of these polarization effects have been derived by Krüger[23] and are repeated below for convenience. The degree of circular polarization is given simply by

$$P=2\mathfrak{R}\mathfrak{F},$$

where $P=+1$ would mean pure right-hand (i.e., angular momentum in the direction of propagation) circularly polarized radiation with the phase convention of Eq. (24). The beta-gamma angular correlation $W(\theta)=1+A\cos\theta+\cdots$ in a transition $J\xrightarrow{\beta}J'\xrightarrow{\gamma}J''$ gives

$$A=\frac{v}{c}\cdot\frac{1-2s[J'(J'+1)]^{1/2}}{1+s^2}\cdot\left[\frac{J'(J'+1)-J(J+1)+2}{[3J'(J'+1)]^{1/2}}\right]\cdot F_1(L,\tilde{L},J'',J')\mathfrak{R}\mathfrak{F}$$

and $s=C_V\int 1/C_A\int\boldsymbol{\sigma}$ for allowed transitions. The coefficients F_1 are tabulated[42,43] by several authors. A pure allowed Fermi transition does not change the magnetic substates and therefore cannot lead to a polarization of the nucleus after beta decay. This can be seen from the above expression by noting that $A\rightarrow 0$ as $s\rightarrow\infty$. The regular transition has multipolarity L and the irregular transition has multipolarity $\tilde{L}$.

B. Alpha Decay

Alpha decay of the type $J^\pi\rightarrow 0^{\pi'}$ with $\pi'=(-1)^{J+1}\pi$ (e.g., $0^-\rightarrow 0^+$, $1^+\rightarrow 0^+$, etc.) cannot occur if parity is conserved. In principle, decay could go via emission of an alpha plus a photon; however, it should be possible to distinguish a line spectrum of alpha particles, whose energy can often be predicted with great accuracy, from the broad spectrum resulting from this mixed decay. It is necessary that either the initial or final state have zero spin since a transition such as $2^+\rightarrow 1^+$, although forbidden to p-wave alpha particles, can occur via d wave, and alpha-decay rates are relatively insensitive to the angular momentum change. Thus we will only be interested here in even-A nuclei.

The width for alpha decay is usually[44,45] written

$$\Gamma_l=2P_l\gamma_l^2,$$

where P_l is the penetration factor and equals $kR/[F_l^2(R)+G_l^2(R)]$ in the notation of Block *et al.*[46]

[42] A. H. Wapstra, G. J. Nijgh, and R. Van Lieshout, *Nuclear Spectroscopy Tables* (North-Holland Publishing Company, Amsterdam, 1959).
[43] K. Alder, B. Stech, and A. Winther, Phys. Rev. **107**, 728 (1957).
[44] R. G. Thomas, Progr. Theoret. Phys. (Kyoto) **12**, 253 (1954).
[45] A. M. Lane, Rev. Mod. Phys. **32**, 519 (1960).

The nuclear structure effects are contained in γ_l which is given by

$$\gamma_l=(2M_rR)^{1/2}\int_S \varphi_l^* X dS,$$

where φ_l is the wave function of an outgoing l-wave alpha particle plus the daughter nuclear state, while X is the parent nuclear state. The integration is over all the internal coordinates of the alpha particle plus daughter final state and integration of their relative coordinates over the "interaction surface" which is simply a sphere of radius R. The quantity R is usually taken to be the Coulomb radius if the initial state is spherically symmetric.

Rather than solving for γ_l, we will try to extract from γ_l the features dependent on the parities. Thus we write

$$\gamma_l=\gamma_l^0\langle\alpha_f|\alpha_i\rangle\langle\psi_f|\psi_i\rangle,$$

where $\langle\alpha_f|$ is the wave function of the actual outgoing alpha particle, while $|\alpha_i\rangle$ is the zero angular momentum state of the four nucleons having center of mass a distance R from the center of mass of the $A-4$ nucleons that will comprise the daughter nucleus, denoted by $|\psi_i\rangle$. The physical daughter nucleus is $\langle\psi_f|$. The quantity γ_l^0 then describes the amplitude for the product nucleus to be in this state of near separation. It will be most convenient here to assume the normalization $\langle\alpha_f|\alpha_i\rangle=\langle\psi_f|\psi_i\rangle=1$ which gives $\gamma_l=\gamma_l^0$ for allowed alpha decay. To "turn on" the parity nonconserving interaction, we then have simply

$$\tilde{\gamma}_l=\gamma_l^0\langle\tilde{\alpha}_f|\tilde{\alpha}_i\rangle\langle\tilde{\psi}_f|\tilde{\psi}_i\rangle,$$

with γ_l^0 essentially unchanged. Wave functions with a tilde refer to the parity impure states, and a prime is employed to indicate just the irregular parity amplitude. Thus $\tilde{\psi}_k=\psi_k+\psi_k'$, where ψ_k is the unperturbed state with quantum numbers k.

To be definite, consider the hypothetical case of a 0^- state in the parent nucleus, unstable energetically to decay to the ground state (0^+) of the daughter, then

$$\begin{aligned}\tilde{\gamma}_0(\text{parent})&=\gamma_0^0[\langle 0^+|0'^-\rangle_{\text{alpha}}\langle 0^+|0^+\rangle_{\text{daughter}}\\&\quad+\langle 0^+|0^+\rangle_{\text{alpha}}\langle 0^+|0'^-\rangle_{\text{daughter}}]\\&\approx\gamma_0^0[\langle 0^+|0'^-\rangle_{\text{alpha}}+\langle 0^+|0'^-\rangle_{\text{daughter}}].\end{aligned}\quad(26)$$

Insofar as the parity nonconserving forces *among* the nucleons of the daughter (alpha) nucleus are concerned, the second (first) term on the right-hand side of (26) is identically zero. The state with nominal parity (+) and the state with nominal parity (−) are distinct solutions of the Hamiltonian including the parity nonconserving forces, and therefore are exactly orthogonal. This need not mean that (26) vanishes, it merely indicates that part of the parity nonconserving interaction cannot contribute to the decay. The contribution from the parity nonconserving forces *between* the "alpha" and the "daughter" nucleus has not been included, and the "alpha" induces a certain amount of irregular parity amplitude in the "daughter" wave function and vice versa. The single-particle approximation then gives

$$\langle 0^+|0'^-\rangle_{\text{daughter}}=\frac{4}{A}MG'\langle 0^+|\sum_i \boldsymbol{\sigma}_i\cdot\mathbf{r}_i|0^-\rangle,$$

taking the average density of the "alpha" particle seen by each nucleon of the "daughter" to be $4/A$ of the average nuclear density. Only one nucleon of the "daughter" is assumed to be in the irregular parity state, an approximation equivalent to assuming the normalization $\langle\psi_f|\psi_i\rangle=1$ for ordinary alpha decay. The equivalent approximations for the influence of the "daughter" on the "alpha" then give altogether

$$\tilde{\gamma}_l=\mathfrak{F}\gamma_l^0.\quad(27)$$

The quantity γ_l^0 will not be computed accurately here, since this is a problem central to all theories of alpha decay and not special to parity nonconservation considerations. Instead, the quantity γ_l^0 may be estimated, within an order of magnitude, to be

$$\gamma_l^0\approx\bar{\theta}\gamma_W,$$

where γ_W is the Wigner limit $(3/2M_rR^2)^{1/2}$ and $\bar{\theta}$ is the average dimensionless reduced width, equal to about 0.1 for light nuclei and 0.2 for heavy nuclei. This gives

$$\begin{aligned}\tilde{\gamma}^2&\approx 0.8\times10^{-10}\text{ eV};\ A=16\quad(\mathfrak{F}^2\approx1.0\times10^{-13})\\&\approx0.9\times10^{-9}\text{ eV};\ A=160\quad(\mathfrak{F}^2\approx4.5\times10^{-13}\\&\qquad\text{average of extra proton and extra neutron}).\end{aligned}$$

Another consideration here is the effect of the parity *conserving* "alpha-daughter" force. Including such an interaction vitiates the orthogonality argument, since the initial and final particles are no longer states of the same internal Hamiltonian. It is, however, consistent with our model to ignore this interaction, assuming the latter to be well approximated by a simple scalar force between the mass centers of the "alpha" and the "daughter," and therefore not influencing the internal motions.

The parity-forbidden alpha decay will always be in competition with other decay modes. The even-even nuclei, having 0^+ ground states, and therefore not parity forbidden to alpha decay, are not of interest, leaving the odd-odd nuclei which are almost all unstable to beta-decay processes (exceptions are H^2, Li^6, B^{10}, and N^{14}). The nuclei, unstable energetically to alpha decay, in which either the daughter or parent spin is zero (always 0^-) are listed in Table I. Ideally, the spin-zero nucleus should be the parent, otherwise branching to excited states usually results in the parity forbidden alpha decay competing with the allowed alpha decay and Eq. (27) gives branching fractions of the order $\mathfrak{F}^2\approx10^{-12}$. Having the spin-zero nucleus as parent will

TABLE I. Alpha decays involving 0^- odd-odd nuclei.

Nucleus		Daughter/Parent		J^π	$-Q_\alpha$(MeV)	$\tau_{1/2}$(sec)	Expected branching
$_{67}Ho^{166}$		$_{65}Tb^{162}$		...	0.046	9.7(4)	
	←		$_{69}Tm^{170}$	1^-	0.769	1.1(7)	1.7(−69)
$_{71}Lu^{170}$		$_{69}Tm^{166}$		(2^-)	2.6	1.7(5)	
			$_{73}Ta^{174}$	...	3.3	4.7(3)	
$_{79}Au^{200}$		$_{77}Ir^{196}$		...	1.0	2.9(3)	
			$_{81}Tl^{204}$	2^-	1.0	1.2(8)	
$_{81}Tl^{206}$		$_{79}Au^{202}$		...	0.342	2.6(2)	
	←		$_{83}Bi^{210}$	1^-	4.946	4.3(5)	5.3(−13)
$_{81}Tl^{210}$		($_{79}Au^{206}$)		...	3.2	2.5(2)	
			$_{83}Bi^{214}$	...	5.599	1.2(3)	
$_{93}Np^{236}$	→	$_{91}Pa^{232}$		1^-	5.14	7.9(4)	3.6(−21)
			$_{95}Am^{240}$	...	5.77	1.8(5)	
$_{95}Am^{244}$	→	$_{93}Np^{240}$		1^-	5.34	1.6(3)	7.5(−21)
			$_{97}Bk^{248}$	...	5.51	7.0(4)	
$_{87}Fr^{224}$			$_{89}Ac^{228}$	2^-	4.61	2.2(4)	
$_{91}Pa^{234}$		$_{89}Ac^{230}$		...	4.20	7.1(1)	
$_{93}Np^{234}$		$_{91}Pa^{230}$		1^-	5.43	3.8(5)	
$_{95}Am^{238}$		$_{93}Np^{234}$		...	6.01	7.2(3)	

[a] The listed half-lives refer to the parent nucleus in the proposed alpha decay, the instability usually being due to beta-decay processes except for Bi^{210} and Bi^{214} which alpha-decay to excited states of Tl^{206} and Tl^{210}. The arrows indicate transitions that satisfy all the requirements for parity-forbidden alpha instability. The last four nuclei in the "nucleus" column do not have measured spins. These are listed because no alpha activity has been observed, and it is therefore possible that parity forbids the transition. Some of the nuclei here assigned to 0^- are themselves uncertain. The half-life and Q values may not be the latest or most accurate values, in some cases being estimated from a semiempirical mass formula, [P. A. Seeger, Nucl. Phys. **25**, 1 (1961)], and this table should not be regarded as a reference for these quantities. An entry such as 9.7(4) indicates 9.7×10^4. The branching fractions were estimated from the reduced widths of neighboring even-even transitions.

not necessarily prevent the possibility of branching, but often the decay to the low-lying levels is also parity forbidden.

Decay from excited states would seem to have all the above difficulties and have additionally to compete with gamma decay. However, a high-spin isomeric state can be virtually stable to both beta and gamma decay while unstable to alpha decay, since the spin change does not play a very important role in the alpha decay. On the other hand, the appearance of low-lying states invites regular alpha decay to these states in competition to the forbidden ground-state decay. An example is the isomeric 9^- state of Bi^{210} which almost entirely decays to excited states of Tl^{206}. From an experimental standpoint, it is undesirable to have 10^{12} alpha particles to scatter about when trying to detect a single rare one having a not very different energy.

In the light nuclei, where one often finds large gaps between the first excited and ground states the branching discussed above should be less serious. Table II illustrates the excited nuclear states of self-conjugate nuclei known to be unstable to parity-forbidden alpha decay. Many of these states have been investigated[2–9] and the upper limits on $\mathfrak{F}$ are included in Table II. Undoubtedly, other examples exist (S^{32} has several uncertain assignments); however, beyond F^{18} the odd-odd self-conjugate nuclei become proton unstable before they become alpha unstable, and the even-even self-conjugate nuclei become beta-decay unstable above $A=40$ and exceedingly difficult to produce above $A=60$. It is not necessary that the nuclei be self-conjugate, but the $E1$ and $M1$ transitions tend to be weak for self-conjugate nuclei and thereby would give less competition to a parity-forbidden alpha decay.

C. Beta Decay

No new effects appear to be introduced by the parity nonconserving force. Essentially, the nuclear parity nonconserving effects generate axial vector couplings from the polar vector interaction and vice versa, but both are present anyway. To display the *form* of the additional couplings, the single-particle approximation will be used first, and then the general result may be written down by inspection. The transition operators for beta decay are simply 1, γ_5, $\boldsymbol{\sigma}$, and $\boldsymbol{\alpha}$ which are further coupled to the retardation expansion of the outgoing lepton wave functions to give the transition operators for "forbidden" transitions. In this abbreviated notation, $\boldsymbol{\sigma}$ represents the operator $\sum_i \boldsymbol{\sigma}_i\tau_\pm{}^i$; the sum extends over all nucleons.

The new operators generated by Eq. (22) are

$$i[1,\boldsymbol{\sigma}\cdot\mathbf{r}]=0,$$
$$i[\gamma_5,\boldsymbol{\sigma}\cdot\mathbf{r}]=0,$$
$$i[\boldsymbol{\sigma},\boldsymbol{\sigma}\cdot\mathbf{r}]=2(\boldsymbol{\sigma}\times\mathbf{r}),$$
$$i[\boldsymbol{\alpha},\boldsymbol{\sigma}\cdot\mathbf{r}]=i[-i\gamma_5\boldsymbol{\sigma},\boldsymbol{\sigma}\cdot\mathbf{r}]=-2i\gamma_5(\boldsymbol{\sigma}\times\mathbf{r}),$$

which give the replacements

$$1\rightarrow 1 \quad \boldsymbol{\sigma}\rightarrow\boldsymbol{\sigma}-2MG''(\boldsymbol{\sigma}\times\mathbf{r}),$$
$$\gamma_5\rightarrow\gamma_5 \quad \boldsymbol{\alpha}\rightarrow\boldsymbol{\alpha}+2G''\mathbf{l}.$$

Our instructions are now to use the parity pure wave functions, and for example an allowed axial vector transition goes by the regular part of $\boldsymbol{\sigma}$ and the irregular part of $\boldsymbol{\alpha}$. Thus it is more useful to write the replacements as

$$1\Rightarrow 1,\quad \boldsymbol{\sigma}\Rightarrow\boldsymbol{\sigma}+2G''\mathbf{l},$$
$$\gamma_5\Rightarrow\gamma_5,\quad \boldsymbol{\alpha}\Rightarrow\boldsymbol{\alpha}-2MG''(\boldsymbol{\sigma}\times\mathbf{r}). \tag{28}$$

TABLE II. Alpha-decay unstable (parity forbidden) excited states of light nuclei. The entry $\mathfrak{F}^2_{\rm expt}$ refers to the upper limit imposed by experiment. In order to deduce $\mathfrak{F}^2_{\rm expt}$, it is always necessary to estimate the irregular alpha-decay width Γ_α' and often necessary to estimate the width for competing decay modes, here designated Γ_γ. The values assumed in the references indicated have therefore been included for comparison. Only states stable to particle decay (n, p, and d) have been included due to the generally large competing widths for such processes. Consequently, the work of Refs. 2 and 3 does not appear in this table. Equation (23) gives $\mathfrak{F}_{\rm theo}$.

Nucleus	(J^π,T)	$\rightarrow (J^\pi,T)$	E(MeV)	$-Q_\alpha$(MeV)	Ref.	Γ_γ(eV)	Γ_α'(eV)	$\mathfrak{F}^2_{\rm expt}$	$\mathfrak{F}^2_{\rm theo}$
${}_3$Li^6	0^+,1	1^+,0	3.560	2.089	...	...	...	...	4.9(−14)
${}_8$O^{16}	2^-,0	0^+,0	8.88	1.72	5	3(−3)	6(2)	1.3(−11)	1.0(−13)
${}_8$O^{16}	2^-,0	0^+,0	8.88	1.72	5	3(−3)	6(3)	1.3(−12)	1.0(−13)
					6	2(−3)	6.7(3)	7(−12)	1.0(−13)
					7	3(−3)	3(3)	1.3(−10)	1.0(−13)
					8	3(−3)	3(3)	2(−12)	1.0(−13)
					9	1(−3)	2.8(2)	2.4(−10)	1.0(−13)
${}_8$O^{16}	0^-,0	0^+,0	10.95	3.79	4	1.8(−1)	2(2)	2(−9)	1.0(−13)
${}_8$O^{16}	3^+,0	0^+,0	11.08	3.92	...	...	...	...	1.0(−13)
${}_{10}$Ne^{20}	2^-,0	0^+,0	4.97	0.24	...	...	...	...	1.1(−13)
	(4^-),0	0^+,0	7.03	2.30	...	...	...	...	1.1(−13)
${}_{14}$Si^{28}	(1^+,1)	0^+,0	10.71	0.72	...	...	...	...	1.4(−13)

Since S commutes with the spatial coordinates, the replacements (28) are then made in every term of the retardation expansion. This means that no new angular correlations, spectrum shape corrections, etc., will appear, and the factors already present will merely be changed slightly (order G) in magnitude.

There is very little hope of distinguishing $\boldsymbol{\sigma}$ from $\boldsymbol{\sigma}+2G''\mathbf{l}$: only if one knew that $\langle\boldsymbol{\sigma}\rangle=0$ could one then decide whether $2G''\langle\mathbf{l}\rangle$ was being detected.

In the more general situation, an operator S must exist such that $\bar{\psi}=e^{iS}\psi$; hence Eq. (28) becomes, to order S,

$$1\Rightarrow 1+i[\gamma_5,S],$$
$$\gamma_5\Rightarrow\gamma_5+i[1,S],$$
$$\boldsymbol{\sigma}\Rightarrow\boldsymbol{\sigma}+i[\boldsymbol{\alpha},S],$$
$$\boldsymbol{\alpha}\Rightarrow\boldsymbol{\alpha}+i[\boldsymbol{\sigma},S],$$

and the possible noncommutivity of S with the coordinate terms of the retardation expansion will not introduce any new dependence on the lepton momenta. The new operators will additionally have new isotopic spin dependences, as mentioned briefly in Sec. IIIA, but this in no way changes the above argument.

D. Gamma Decay

If the spin-parity change in a nuclear deexcitation allows emission of a photon of given multipolarity, say EL, then parity admixing in general allows the multipolarity ML to also be emitted. Experimentally, the irregular radiation must be detected indirectly, as, for example, from the interference of the two amplitudes to give a slight ellipticity to the polarization of the radiation. As suggested in Sec. IIIB, this effect may be greatest for transitions of the form $EL(ML)$ and weakest for $ML(EL)$. Intuitively, we might argue that the tendency for the nucleons to correlate their spin and momentum will not be manifested externally unless the correlation *changes* in the transition from initial to final states. In the weak electric transition, the correlation changes only in that the initial and final wave functions do not, in general, have quite the same admixing. In the single-particle approximation, they have equal admixing and the weak EL vanishes identically. On the other hand, the magnetic transitions, e.g., spin-flip, reverse the spin direction and give a maximal change in correlation.

The magnitude of the circular polarization is so small, of order $\mathfrak{F}$, that direct experimental verification is at present unlikely. One must search for situations in which the parity-allowed transition is greatly inhibited, so that the weak parity-forbidden transition will show up more, provided it is not also inhibited. The great difficulty of this program is that one cannot be at all sure, given a strongly inhibited EL transition, that the weak ML is not inhibited for the same reason. For example, the 56-keV transition from the isomeric state of Hf^{180} ($8^-\rightarrow 8^+$) is enormously inhibited (10^{-16} of the Wigner width) and *a priori* looks like a perfect example. However, the 8^+ state belongs to a $K=0$ band while the 8^- is thought to be the lowest member of a $K=8$ band. The selection rule[47,48] $\Delta K=0$, 1 for dipole transitions is then strongly violated, explaining the inhibition, and this selection rule applies equally to the $M1$ transitions.

What is usually done in practice is to choose a nucleus or group of nuclei in which the $E1$ transitions, say, are inhibited but the neighboring $M1$ transitions are not. In the absence of any information on the structure of the nuclear states, this is probably about as good as one can do. It is therefore advantageous to study nuclei about which one possesses some knowledge of the structure, in this case Eq. (22) may be employed to generate the appropriate operators for the irregular transitions and the transition amplitudes may then be computed. It is not usually necessary to compute the inhibited regular transitions since this can be *measured*

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[47] G. Alaga, Nucl. Phys. 4, 625 (1957).
[48] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter 1, No. 8 (1959).

experimentally, while the amplitude of the irregular transition (which should be uninhibited if the case is to be promising experimentally) can be computed with some confidence. In Appendix A it is pointed out that Eq. (14′) does not contribute to the electromagnetic coupling in the single-particle approximation, thus the electromagnetic transition operators are unchanged by addition of the parity nonconserving interaction (14). Gauge invariance[49] guarantees in any event that the EL operator is directly proportioned to the time derivative of the corresponding static multipole moment operator. Thus the dipole transition operator is $de\mathbf{r}/dt = i[H,e\mathbf{r}] = i\omega e\mathbf{r}$, where ω is the transition energy. This result is easily verified for the Hamiltonian (20) by using the replacement $\mathbf{p}\rightarrow\mathbf{p}-e\mathbf{A}$ to introduce the electromagnetic interaction and comparing the result (in the dipole approximation) with $i[H,e\mathbf{r}]$. In transforming via (21) to the unperturbed system, it makes no difference which form of the transition operation is employed since $e^{-iS}i[H,e\mathbf{r}]e^{iS}=i[H_0,e\mathbf{r}]=i\omega e\mathbf{r}$ in the unperturbed system also. The numerical quantity ω is unchanged to order G^2. The $M1$ operator $(e/2M)\times(3/4\pi)^{1/2}(g_\sigma\boldsymbol{\sigma}+g_l\mathbf{l})_z$ gives from Eq. (22) the $\tilde{M}1$ operator $(e/2M)(3/4\pi)^{1/2}(2g_\sigma-g_l)(i\boldsymbol{\sigma}\times\mathbf{r})_z$. The $E1(M1)$ transitions then give

$$\mathcal{R}_{E1(M1)}=\left(\frac{2g_\sigma-g_l}{2MR}\right)\langle\|i\boldsymbol{\sigma}\times\mathbf{r}\|\rangle/\langle\|\mathbf{r}\|\rangle. \tag{29}$$

Similar expressions obtain in the single-particle approximation for higher $EL(ML)$ multipolarities from the replacements

$$\boldsymbol{\sigma}\rightarrow 2i\boldsymbol{\sigma}\times\mathbf{r},\quad \mathbf{l}\rightarrow -i\boldsymbol{\sigma}\times\mathbf{r} \tag{30}$$

in the appropriate ML transition operator. It might be noted here that vanishing of $(\|e\mathbf{r}\|)$ will not guarantee an arbitrarily large value of $\mathcal{R}_{E1(M1)}$, since the small magnetic effects also give an additional contribution $-i\omega g_\sigma e(\|\boldsymbol{\sigma}\times\mathbf{r}\|)/4M$ to the $E1$ transition operator which gives a value for $\mathcal{R}_{E1(M1)}=-2(2g_\sigma-g_l)/g_\sigma\omega R$. This does not constitute a limit since the usual dipole contribution $(\|e\mathbf{r}\|)$ can interfere with the magnetic contribution.

As discussed in Sec. IIIB, the $ML(EL)$ transitions give $\mathcal{R}_{M1(E1)}=0$ in the single-particle approximation. This does not appear to be a fundamental result, however, and simply suggests that this type of transition is less promising for demonstration of parity nonconserving effects. Nevertheless, it is important to know the form of the operator for these $\tilde{E}L$ transitions if favorable experiments are to be selected to test this conjecture. Assuming that $[S,\mathbf{r}]\neq 0$ when more realistic Hamiltonians are used, e.g., including the spin-orbit force, we then have from Eq. (19) an operator of the form $\frac{1}{2}\{\gamma(r),(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}\}_+$, where $\gamma(r)$ is a scalar operator. Approximating γ to be a constant then gives $\gamma(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}$, and we adopt the form $\tilde{E}L=\gamma(\boldsymbol{\sigma}\cdot\mathbf{r})EL$, thus

$$\mathcal{R}_{M1(E1)}\approx\frac{2M\gamma(\|(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}\|)}{R(\|g_\sigma\boldsymbol{\sigma}+g_l\mathbf{l}\|)}. \tag{31}$$

The tendency for γ to vanish will be compensated somewhat by the relatively greater strength of the EL transitions to ML which otherwise favors $ML(EL)$ over $EL(ML)$ transitions as seen by the factor $MR\sim 6A^{1/3}$ in Eqs. (29) and (31).

A promising theoretical situation obtains in nuclei where the closing of shells and/or strong pairing forces seem to allow, as a sensible approximation, the separation of the nucleus into a single nucleon coupled to an inert core.

Spherical Nuclei

If the core is spherically symmetric, the extra-core nucleon will have a definite l and j, and the quantity $\mathcal{R}$ may then be computed with some confidence for transitions among the possible (l,j) states. Situations of interest for detection of parity nonconservation are where the regular transition is forbidden and the irregular transition is allowed. For the $EL(ML)$ single-particle transitions this never occurs, since the transition is allowed if $|j_f-j_i|\leq L\leq|j_f+j_i|$ and the parity changes, conditions that $\tilde{M}L$ must also satisfy.

The value of $\mathcal{R}_{E1(M1)}$ is given for the possible single-particle $E1$ transitions in Eq. (32) below. The radial parts of the transition amplitudes cancel in $\mathcal{R}_{E1(M1)}$ in the single-particle approximation.

$$\begin{aligned}\mathcal{R}_{E1(M1)}&=2(l+1)\cdot(2g_\sigma-g_l)/2MR\\ &\qquad (l,\,l+\tfrac{1}{2})\rightarrow(l+1,\,l+\tfrac{1}{2})\\ &=\pm\tfrac{1}{4}\cdot(2g_\sigma-g_l)/2MR\\ &\qquad (l,\,l\pm\tfrac{1}{2})\rightarrow(l+1,\,l+1\pm\tfrac{1}{2}).\end{aligned} \tag{32}$$

The $E1$ transition $(l,l+\frac{1}{2})\rightarrow(l+1,l+\frac{1}{2})$ is relatively weak since the spin direction changes (in the classical limit), accounting for the larger $\mathcal{R}$ value.

In the $M1(E1)$ transitions we obtain similarly, using expression (31),

$$\begin{aligned}\mathcal{R}_{M1(E1)}&=-2\gamma' &&(l,\,l+\tfrac{1}{2})\rightarrow(l,\,l-\tfrac{1}{2})\\ &=\mp\gamma'/(2l+1\pm 2) &&(l,\,l\pm\tfrac{1}{2})\rightarrow(l,\,l\pm\tfrac{1}{2}),\end{aligned} \tag{33}$$

where

$$\gamma'=2\gamma M\langle r^2\rangle/R(g_\sigma+g_l\langle\|\mathbf{l}\|\rangle/\langle\|\boldsymbol{\sigma}\|\rangle)\sim(7.2A^{1/3}\gamma/g_\sigma).$$

It is relatively straightforward to compute γ for the special case of a harmonic oscillator potential with a spin-orbit force $(a/2)\boldsymbol{\sigma}\cdot\mathbf{l}$. The contribution comes in this case from the spin-orbit contributions to the energy denominators which destroys the otherwise perfect cancellation of the irregular terms. The results are, to first order in $a/\omega_0\sim\frac{1}{3}A^{-1/3}$ where ω_0 is the oscillator level

[49] R. G. Sachs and N. Austern, Phys. Rev. 81, 705 (1951).

spacing,

$$\gamma=3\frac{a}{\omega_0^2R}(l+\tfrac{1}{2}) \qquad (l,l\pm\tfrac{1}{2})\to(l,l\mp\tfrac{1}{2})$$
$$0 \qquad (l,l\pm\tfrac{1}{2})\to(l,l\pm\tfrac{1}{2}) \tag{34}$$
$$=-\frac{a}{\omega_0^2R}\cdot[(\kappa+l+\tfrac{1}{2})(\kappa+l+\tfrac{3}{2})]^{1/2} \qquad (l-1,l-\tfrac{1}{2})\to(l+1,l+\tfrac{1}{2}),$$

where κ is the number of nodes in the radial wave function. The empirical estimates $a\sim 13A^{-2/3}$ MeV and $\omega_0\sim 41A^{-1/3}$ MeV give $a/\omega_0^2R\sim 1.3A^{-1/3}$ and $\gamma'\sim 9.1/g_\sigma$. It must be stressed that these estimates are mainly to indicate the order of magnitude of the corrections involved; the finite size of the nuclear core also gives corrections of magnitude similar to Eq. (34). In the magnetic transitions, a special selection rule does operate. The "$M1$" transition $(l-1,l-\frac{1}{2})\to(l+1,l+\frac{1}{2})$ is, in fact, forbidden since the magnetic moment operator cannot change the orbital angular momentum by 2 units. Such "l-forbidden" magnetic transitions[50–52] occur for every magnetic multipole order. The $E2$ transition ($El+1$, in general) is not forbidden, and, if the $M1$ were rigorously forbidden, we would have $E2(E1)$ interference giving

$$\mathcal{R}_{E2(E1)}=-[(2l-1)(2l+3)/3]^{1/2}\cdot(\gamma/\omega_0^2R^2) \qquad (l-1,l-\tfrac{1}{2})\to(l+1,l+\tfrac{1}{2})$$
$$\sim -24lA^{-1/3}.$$

In general, the tensor force admixes l with $l\pm 2$ and these l-forbidden transitions are observed to be about two orders of magnitude slower than the Weisskopf estimates.[53] Hence,

$$|\mathcal{R}_{M1(E1)}|\sim 10(\Gamma_w(E1)/\Gamma_w(M1))^{1/2}\sim 18A^{1/3} \qquad (l-1,l-\tfrac{1}{2})\to(l+1,l+\tfrac{1}{2}).$$

When more than one particle contributes to the electromagnetic decay, as would be expected in nuclei whose states cannot be reasonably separated into core +extra nucleon, the amplitudes for the regular transition may interfere destructively, while the irregular amplitude interferes constructively. Such interference occurs for example in $T=0\to T=0$ transitions of self-conjugate ($N=Z$) nuclei. Here, the $E1$ transitions (only) are forbidden, except via isotopic spin impurities,[54] and the transition amplitudes are observed to be reduced by factors of about 100. It happens, however, that the $M1$ transitions are also weak[55,56] (by a factor of about 10 in amplitude) and $\mathcal{R}_{E1(M1)}$ is amplified by a factor of about 10 over the estimates of Eq. (32).

Deformed Nuclei

If the core is strongly deformed, the orbital and spin angular momenta of an extra-core particle tend to become decoupled and a greater variety of selection rules can act. In this model,[41,48] the single-particle wave functions are described by $JK\pi[Nn_z\Lambda]$ in the usual notation.[57] The states admixed by any pseudoscalar interaction in the nuclear Hamiltonian will have $\Delta J=\Delta K=0$ and $\Delta\pi$ (yes). Since the parity is given by $(-1)^N$, we must have $\Delta N=$odd, and for the single-particle approximation, $\Delta N=1$. The change in orbital angular momentum of the extra particle must be ± 1 for a single-particle parity-nonconserving scalar potential since the spin operator can at most be a tensor of rank 1 and must couple with the spatial dependence to give a rank-zero tensor; hence, the spatial operator must also be rank 1 and therefore can couple only states differing by one unit of angular momentum. The projection of the orbital angular momentum is then restricted to $|\Delta\Lambda|=1$, 0 requiring $|\Delta n_z|=0$, 1, respectively. These rules may easily be deduced for $\boldsymbol{\sigma}\cdot\mathbf{r}$ from Table I of Ref. 48, and are summarized below in Eq. (35). The above discussion is to emphasize the independence of the selection rules from any specific operator such as $\boldsymbol{\sigma}\cdot\mathbf{r}$.

$$(\sigma_+r_-+\sigma_-r_+)\quad \Delta\Lambda=\pm1,\quad \Delta n_z=0,\quad \Delta N=\pm1,\mp1,$$
$$(\sigma_z z)\quad \Delta\Lambda=0,\quad \Delta n_z=\pm1,\quad \Delta N=\pm1. \tag{35}$$

It has already been mentioned in Sec. IIIB that spherical symmetry is not required in the single-particle approximation.

Suppose a transition involves $\Delta N=2$ while the

TABLE III. Asymptotic selection rules for the regular $E1$ and irregular $M1$ transitions.

ΔK	Multipole	Operator	$\Delta\Lambda$	Δ_{nz}	ΔN
0	$E1$	z	0	±1	±1
	$\tilde{M}1$	$(\sigma_x\pm i\sigma_y)(x\mp iy)$	±1	0	$\pm1,\mp1$
1	$E1$	$(x+iy)$	±1	0	$\pm1,\mp1$
	$\tilde{M}1$	$\sigma_z(x+iy)$	±1	0	$\pm1,\mp1$
	$\tilde{M}1$	$(\sigma_x+i\sigma_y)z$	0	±1	±1

[50] A. Arima, H. Horie, and M. Sano, Progr. Theoret. Phys. (Kyoto) **17**, 567 (1957).
[51] G. M. Bukat, Zh. Eksperim. i Teor. Fiz. **39**, 1716 (1960) [translation: Soviet Phys.—JETP **12**, 1198 (1961)].
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[53] D. H. Wilkinson, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove, (Academic Press Inc., New York and London, 1960), Part B, p. 859.
[54] M. Gell-Mann and V. L. Telegdi, Phys. Rev. **91**, 169 (1953).
[55] G. Morpurgo, Phys. Rev. **110**, 721 (1958).
[56] G. K. Warburton, Phys. Rev. Letters **1**, 68 (1958).
[57] J is the total momentum of the state, K is the projection of the single-particle total angular momentum on the symmetry (z) axis of the deformed core, $\pi=(-1)^N$ is the parity of the state, N is the number of nodes in the single-particle wave function, n_z is the number of nodes on the z axis, and Λ is the projection of the orbital angular momentum on the z axis.

TABLE IV. Asymptotic selection rules for irregular $\tilde{M}L$ transitions.[a]

ΔK	Multipole	Operator	$\Delta\Lambda$	Δn_z	ΔN	Selection rule
0	$\tilde{M}1$	$\sigma_+ r_-$	-1	0	±1	
		$\sigma_- r_+$	1	0	±1	
1		$\sigma_+ z$	0	±1	±1	a
0	$\tilde{M}2$	$\sigma_+ r_- z$	-1	±1	$0, \pm2$	
		$\sigma_- z r_+$	1	±1	$0, \pm2$	
1		$\sigma_+ r_- r_+$	0	0	±2	
		$\sigma_- r_+^2$	2	0	$0, \pm2$	
		$\sigma_+ z^2$	0	$\begin{pmatrix}0\\ \pm2\end{pmatrix}$	$\begin{pmatrix}0\\ \pm2\end{pmatrix}$	
2		$\sigma_+ z r_+$	1	±1	$0, \pm2$	d
0	$\tilde{M}3$	$\sigma_+ r_- z^2$	-1	$\begin{pmatrix}0\\ \pm2\end{pmatrix}$	$\begin{pmatrix}\pm1\\ \pm1,3\end{pmatrix}$	
		$\sigma_- z^2 r_+$	1	$\begin{pmatrix}0\\ \pm2\end{pmatrix}$	$\begin{pmatrix}\pm1\\ \pm1,3\end{pmatrix}$	
		$\sigma_+ r_-^2 r_+$	-1	0	$\pm1,3$	
		$\sigma_- r_- r_+^2$	1	0	$\pm1,3$	
1		$\sigma_+ r_- z r_+$	0	±1	$\pm1, \mp1, \pm3$	
		$\sigma_- z r_+^2$	2	±1	$\pm1, \mp1, \pm3$	
		$\sigma_+ z^3$	0	$\begin{pmatrix}0\\ \pm1\\ \pm3\end{pmatrix}$	$\begin{pmatrix}0\\ \pm1\\ \pm3\end{pmatrix}$	
2		$\sigma_+ r_- r_+^2$	1	0	$\pm1, \pm3$	
		$\sigma_- r_+^3$	3	0	$\pm1, \pm3$	
		$\sigma_+ z^2 r_+$	1	$\begin{pmatrix}0\\ \pm2\end{pmatrix}$	$\begin{pmatrix}\pm1\\ \pm1,3\end{pmatrix}$	
3		$\sigma_+ z r_+$	2	±1	$\pm1, \mp1, \pm3$	f

[a] The selection rules have been chosen such that the irregular transition is allowed while the regular transition is hindered. Thus, an EL transition will be hindered if the selection rule for a $\tilde{M}L$ transition is satisfied. To simplify the tables, ΔK has been taken positive: the operator giving $K\Delta = -|\Delta K|$ is simply generated by interchange of (+) and (−) subscripts. Here $\sigma_\pm = 2^{-1/2}(\sigma_x \pm i\sigma_y)$ and $r_\pm = 2^{-1/2}(x \pm iy)$. Those rules known to apply to observed transitions are denoted under "Selection Rule" and correspond to those listed in Table VII.

TABLE V. Irregular EL transitions.[a]

ΔK	Multipole	Operator	$\Delta\Lambda$	Δn_z	ΔN	Selection rule
0	$\tilde{E}1$	$\sigma_+ r_- z$	-1	±1	$0, \pm2$	c
		$\sigma_- z r_+$	1	±1	$0, \pm2$	c
1		$\sigma_+ r_- r_+$	0	0	±2	
		$\sigma_- r_+^2$	2	0	$0, \pm2$	b
0	$\tilde{E}2$	$\sigma_+ r_- z^2$	-1	±2	$\pm1,3$	e
		$\sigma_- z^2 r_+$	1	±2	$\pm1,3$	e
		$\sigma_+ r_-^2 r_+$	-1	0	±2	
		$\sigma_- r_- r_+^2$	1	0	±3	
1		$\sigma_+ r_- z r_+$	0	±1	$\mp1, \pm3$	
		$\sigma_- z r_+^2$	2	±1	$\pm1, \mp1, \pm3$	
2		$\sigma_+ r_- r_+^2$	1	0	±3	
		$\sigma_- r_+^3$	3	0	$\pm1,3$	
0	$\tilde{E}3$	$\sigma_+ r_- z^3$	-1	±3	$\pm2,4$	
		$\sigma_- z^3 r_+$	1	±3	$\pm2,4$	
		$\sigma_+ r_-^2 z r_+$	-1	±1	±4	
		$\sigma_- r_- z r_+^2$	1	±1	±4	
1		$\sigma_+ r_- z^2 r_+$	0	±2	$0, \pm4$	
		$\sigma_+ r_-^2 r_+^2$	0	0	±4	
		$\sigma_- z^2 r_+^2$	2	±2	$0, \pm2, \pm4$	
		$\sigma_- r_- r_+^2$	2	0	±4	
2		$\sigma_+ r_- z r_+^2$	1	±1	$\mp2, \pm4$	
		$\sigma_- z r_+^3$	3	±1	$0, \pm2, \mp2, \pm4$	
3		$\sigma_+ r_- r_+^3$	2	0	±4	
		$\sigma_- r_+^4$	4	0	$0, \pm2, \pm4$	g

[a] These selection rules for the $\tilde{E}L$ transitions hinder both the ML and the $EL+1$ transitions.

selection rules allow at most $\Delta N = 1$. Then we will have a "hindered" transition, forbidden if the asymptotic quantum numbers were good. For convenience, let us refer to such a situation as having a single "order" of hinderance and not distinguish among hinderance in Λ, n_z, or N but merely sum together and quote a net order of hinderance. Then an allowed transition would match its selection rule completely and have zero order of hinderance. For each order of hinderance, a transition is found[48] to be reduced in amplitude by about a factor of 10^2, depending on the deformation of the core. From the selection rules in Eq. (35), we see that the parity irregular transitions can be less forbidden than the regular transitions by two orders of hinderance. To make this clear, Table III gives the selection rules for the $E1$ and $\tilde{M}1$ transitions, i.e., the operators $\mathbf{r}$ and $\boldsymbol{\sigma}\times\mathbf{r}$. We see that among possible transitions, one with $\Delta K[\Delta N \Delta n_z \Delta\Lambda] = 1[101]$ is allowed both for the regular and irregular transition, and would therefore give no particular enhancement to $\mathcal{R}_{E1(M1)}$ while the transition $0[011]$ should give $\mathcal{R}_{E1(M1)}$ much less than unity, and the transition $0[101]$ should give $\mathcal{R}_{E1(M1)}$ much larger than unity. The latter are of interest in this paper and are listed up to electromagnetic multipole order 3 in Tables IV, V, and VI. For the $\tilde{E}L$ transitions in Table V, we have used the form (31) and required both the ML *and* $EL+1$ to be relatively hindered.

A note of caution: The selection rules are, in general, incomplete since those rules which also permit an unhindered regular transition have been deleted. For example, the $\tilde{E}1$ operator $\sigma_+ r_- r_+$ has the same selection rules as the $M1$ operator σ_+ except for $\Delta N = \pm2$, and this alone has been listed in Table V. Note also that the quantum numbers of a state are in the order $(N n_z \Lambda)$, while it is customary to tabulate the changes in the opposite order as is done in Tables IV, V, and VI.

Table VII lists the intrinsic states, identified in deformed nuclei, whose irregular decay only is allowed by the selection rules of Table IV and V. We will here examine in detail mainly the dipole transitions, since it is often difficult to observe the higher multipole radiation. For example, the 424-keV state of Tm^{171} decays almost entirely to the spin-$\frac{5}{2}$ and -$\frac{7}{2}$ members of the ground-state rotational band ($\frac{7}{2}-[523] \to \frac{1}{2}+[411]$) via the K-forbidden but nevertheless more intense $E1$ and $M2$ radiation in preference to the

TABLE VI. Irregular EL transitions in magnetic orbit flip transitions.

ΔK	Multipole	Operator	$\Delta\Lambda$	Δn_z	ΔN
1	$\tilde{E}1$	$\sigma_z z r_+$	1	±1	$0, \pm2$
2	$\tilde{E}2$	$\sigma_z z r_+^2$	2	±1	$\pm1, \mp1, \pm3$
3	$\tilde{E}3$	$\sigma_z z r_+^3$	3	±1	$0, \pm2, \mp2, \pm4$
4	$\tilde{E}4$	$\sigma_z z r_+^4$	4	±1	$\pm1, \mp1, \pm3, \mp3, \pm5$

TABLE VII. Allowed irregular-hindered regular electromagnetic transitions.[a]

Multipole	Selection rule	Orbit assignments	Nuclei
$\tilde{M}1$	*a*	$\frac{5}{2}+[413]—\frac{7}{2}-[523]$	Tb^{161}
		$\frac{5}{2}-[523]—\frac{7}{2}+[633]$	Er^{167} Np^{235} Am^{245}
		$\frac{7}{2}+[404]—\frac{9}{2}-[514]$	$Lu^{173,175,177}$ $Ta^{179,181}$
		$\frac{7}{2}-[514]—\frac{9}{2}+[624]$	$Hf^{177,179}$
		$\frac{5}{2}+[633]—\frac{7}{2}-[743]$	U^{233} Pu^{239}
		$\frac{7}{2}+[624]—\frac{9}{2}-[734]$	Cm^{245}
$\tilde{E}1$	*b*	$\frac{3}{2}+[411]—\frac{5}{2}+[413]$	Eu^{153} Tb^{161}
		$\frac{3}{2}-[521]—\frac{5}{2}-[523]$	Dy^{161} Er^{165} Np^{237} Am^{23}
		$\frac{5}{2}+[402]—\frac{7}{2}+[404]$	Tm^{171} Lu^{175} Ta^{181}
		$\frac{1}{2}-[510]—\frac{3}{2}-[512]$	$W^{181,183}$
		$\frac{3}{2}+[631]—\frac{5}{2}+[633]$	U^{233}
		$\frac{5}{2}+[622]—\frac{7}{2}+[624]$	Pu^{241} Cm^{245}
	c	$\frac{7}{2}-[503]—\frac{7}{2}-[514]$	Hf^{177}
		$\frac{1}{2}+[400]—\frac{1}{2}+[411]$	Re^{185}
		$\frac{5}{2}+[622]—\frac{5}{2}+[633]$	Pu^{239}
$\tilde{M}2$	*d*	$\frac{5}{2}+[402]—\frac{1}{2}+[411]$	Tm^{171} Lu^{175} Ta^{181} W^{183} Re^{185}
		$\frac{7}{2}-[503]—\frac{3}{2}-[512]$	W^{181}
		$\frac{5}{2}+[622]—\frac{1}{2}+[631]$	U^{237} Pu^{239} Cm^{245}
$\tilde{E}2$	*e*	$\frac{5}{2}+[642]—\frac{5}{2}-[523]$	Dy^{161} $Np^{235,237,239}$ $Am^{239,241,245}$
		$\frac{7}{2}-[523]—\frac{7}{2}+[404]$	$Tm^{169,171}$
		$\frac{7}{2}+[633]—\frac{7}{2}-[514]$	Yb^{171} Hf^{177}
$\tilde{M}3$	*f*	$\frac{7}{2}-[523]—\frac{1}{2}+[411]$	Ho^{163} Tm^{169}
		$\frac{7}{2}+[633]—\frac{1}{2}-[521]$	Dy^{169} Er^{167} Yb^{171}
		$\frac{9}{2}+[624]—\frac{3}{2}-[512]$	W^{181}
		$\frac{7}{2}-[743]—\frac{1}{2}+[631]$	U^{233} $Pu^{237,239}$
$\tilde{E}3$	*g*	$\frac{7}{2}-[514]—\frac{1}{2}-[510]$	Hf^{179} W^{179}

[a] Those transitions are listed which satisfy the selection rules in Tables IV, V, and VI, and are known (Ref. 48) to be present in the low-lying states of deformed nuclei. The list of such nuclei is probably less complete than the possible orbit transitions. Only seven of the rules of Tables IV, V, and VI seem to apply to low-lying states of actual nuclei and these are denoted in the column "Selection Rule" in this and the above tables. The orbit assignments are in the notation (Ref. 57) $K\pi[Nn_z\Lambda]$.

hindered $E3$. Furthermore, the higher multipole transitions do not seem to be as strongly hindered as are the $E1$, with a few exceptions discussed below.

Experimentally, the $E1$ decay rates are often found to be surprisingly slow compared to single-particle estimates. Of those listed in Table VII, the $\frac{9}{2}-[514]\rightarrow\frac{7}{2}+[404]$ and $\frac{9}{2}+[624]\rightarrow\frac{7}{2}-[514]$ transitions are found[58] to be considerably hindered while the irregular $M1$ transition is allowed from Table IV. This hinderance, necessary to accentuate the interference with the irregular $M1$ amplitude must not in turn hinder the irregular $M1$ amplitude. We now briefly consider this point.

The states most accessible for study in the heavy nuclei are the low-lying levels populated by beta decay and the subsequent gamma-ray cascade, if any. For these states, the possible $E1$ transitions *all* violate the asymptotic (i.e., $\Delta\Lambda$, Δn_z, and ΔN) selection rules, despite frequent appearance of states having parity opposite that of the ground state. Let us examine the $\frac{9}{2}-[514]\rightarrow\frac{7}{2}+[404]$ transition. In the limit of very small deformation, the transition becomes $1h_{11/2,9/2}\rightarrow 1g_{7/2,7/2}$ (shell-model notation: L_{jm} where m is the projection of j on a fixed axis), and the $E1$ matrix element must thus vanish for zero deformation as well as large deformation. It need not be surprising then to find that this matrix element never becomes very large for any deformation. Numerical calculations have been made[59] and indeed give very small transition rates. For some transitions the observed rate is still smaller by two orders of magnitude; however, these calculations involve almost complete cancellation of large terms (the cancellation is exact at zero deformation, and the terms themselves approach zero at large deformations), and the model is almost certainly inadequate for quantitative estimates under such circumstances. Admixing among the low-lying states will not particularly enhance a given $E1$ transition if none have appreciable $E1$ transition elements. The higher multipole transitions, having a greater variety of selection rules, are often allowed between low-lying states and admixing of these states conspires to weaken hinderance given by the asymptotic selection rules, as is found empirically. The point of this discussion is to note that the slowness of $E1$ transitions is not in conflict with the Nilsson model, and therefore allowed transitions, being

[58] E. Bashandy and M. S. El-Nesr, Arkiv Fysik 22, 357 (1962).

[59] U. Hauser, K. Runge, and G. Knissel, Nucl. Phys. 27, 632 (1961).

TABLE VIII. Strongly hindered regular $E1$ transitions. These transitions are known to be strongly hindered and in fact obey selection rule "a" of Tables IV–VII. The quantity $t_{1/2}$ refers to the *photon* half-life of the transition. The experimental electric dipole transition rate per second is given by $T(E1)$ while $T(\tilde{M}1)$ is the expected irregular magnetic dipole transition rate calculated as illustrated from Eq. (36) *et seq.* The quantity $|\mathcal{R}_{E1(M1)}|=[T(\tilde{M}1)/T(E1)\mathfrak{F}^2]^{1/2}$ as follows from the definition (25). Estimates from Eq. (23) are listed under $\mathfrak{F}$, and the product $\mathcal{R}\mathfrak{F}$ is that quantity to be used in the formulas at the end of Sec. IVA. The quantity $(\beta_1/\alpha_1)^{1/2}$ refers to the polarization properties of the electron as discussed in Sec. IVE, where β_1 and α_1 are the internal conversion coefficients for the magnetic and electric dipole transitions, respectively.

Nucleus	$(JK)\to(J'K')$	E_γ(keV)	$t_{1/2}$(sec)	$T(E1)$	$T(\tilde{M}1)/\mathfrak{F}^2$	$\lvert\mathcal{R}_{E1(M1)}\rvert$	$\mathfrak{F}$	$\lvert\mathcal{R}\mathfrak{F}\rvert$	$(\beta_1/\alpha_1)^{1/2}$
				Odd proton					
$_{71}$Lu^{173}	$\frac{7}{2}\ \frac{7}{2}$—$\frac{9}{2}\ \frac{9}{2}$[a]	123	8.4(−5)	8.3(3)	2.0(10)	1550[a]	8.0(−7)	1.3(−3)	3.3
$_{71}$Lu^{175}	$\frac{9}{2}\ \frac{9}{2}$—$\frac{7}{2}\ \frac{7}{2}$	396	6.3(−9)	1.1(8)	5.4(11)	70	8.0(−7)	5.7(−5)	1.1[b]
	$\frac{9}{2}$	282	1.1(−8)	6.3(7)	4.4(10)	26	8.0(−7)	2.2(−5)	2.9
	$\frac{11}{2}$	148	9.8(−8)	7.1(6)	6.3(8)	9	8.0(−7)	7.5(−6)	3.2
$_{71}$Lu^{177}	$\frac{9}{2}\ \frac{9}{2}$—$\frac{7}{2}\ \frac{7}{2}$	146	2.8(−7)	2.5(6)	1.8(11)	270	8.0(−7)	2.2(−4)	3.2
	$\frac{9}{2}$	28							5.4[c]
$_{73}$Ta^{181}	$\frac{9}{2}\ \frac{9}{2}$—$\frac{7}{2}\ \frac{7}{2}$	6	6.8(−6)	2.3(3)	2.1(6)	30	8.0(−7)	2.5(−5)	∼34[d]
				Odd neutron					
$_{72}$Hf^{177}	$\frac{9}{2}\ \frac{9}{2}$—$\frac{7}{2}\ \frac{7}{2}$	321	4.9(−8)	1.4(7)	3.7(11)	160	5.5(−7)	8.8(−5)	0.8[e]
	$\frac{9}{2}$	208	7.2(−10)	9.6(8)	2.3(10)	5	5.5(−7)	2.8(−6)	3.1
	$\frac{11}{2}$	71	7.3(−8)	9.6(6)	9.0(7)	3	5.5(−7)	1.7(−6)	3.4
$_{72}$Hf^{179}	$\frac{7}{2}\ \frac{7}{2}$—$\frac{9}{2}\ \frac{9}{2}$	217	<3.5(−7)	>2.0(6)	1.4(11)	<85	5.5(−7)	<4.8(−5)	3.1

[a] J. W. Mihelich *et al.*, Bull. Am. Phys. Soc. **3**, 358 (1958), gives this assignment; however, J. Valentin, Nucl. Phys. **31**, 353 (1961), assigns this state to the 5/2 1/2−[541] orbit, in which case $\Delta K=3$ and not selection rule "a" explains the hinderance. In the latter case, there are no grounds for assigning a large value of $\mathcal{R}$ to this transition.
[b] α_1(expt) ≈6α_1(theo) giving the reduced value listed.
[c] Calculated for L_I capture.
[d] Calculated for M_I capture.
[e] α_1(expt) =12α_1(theo) giving the reduced value listed.

insensitive to detailed knowledge of the single-particle wave functions, can be estimated with some confidence. Of course, should additional factors be shown to contribute, such as a change in the core deformation between initial and final states, the effect on the irregular transition amplitude must also be included. In the single-particle approximation, a change in core deformation would equally hinder regular and irregular transitions.

Table VIII lists the $E1$ transitions among the $\frac{9}{2}\to\frac{7}{2}$ states discussed above, together with the rate of the irregular $M1$ transition and $\mathcal{R}_{E1(M1)}$ computed for the asymptotic wave functions. These transitions are so strongly hindered that the $M2$ admixing may be appreciable and the $M2(M1)$ interference should be considered in these cases (the 396-keV transition in Lu^{175} is about 20% $M2$ and, as a consequence, the coefficient to $\cos\theta_{\beta\gamma}$ in the beta-gamma angular correlation is either $-0.5\mathcal{R}\mathfrak{F}$ or $-0.4\mathcal{R}\mathfrak{F}$, depending on the relative phase of the $E1$ and $M2$ transition amplitudes).

The physical situation here is seen more readily by expanding the $JK\pi[Nn_z\Lambda]$ states in terms of the basis states $|Nl\Lambda\Sigma\rangle$ where l is the orbital angular momentum, the projection of l on the symmetry axis is Λ, and Σ is the same projection of the nucleon intrinsic spin. Note that $K=\Lambda+\Sigma$, $N\geq l$, and $(-1)^N=(-1)^l$. The $\frac{7}{2}+[404]$ state is almost pure[41] $|444-\rangle$ in this notation while the $\frac{9}{2}-[514]$ state is almost pure[48] $|554+\rangle$, thus the $E1$ operator $(x+iy)$ vanishes while the $\tilde{M}1$ operator $z(\sigma_x+i\sigma_y)$ does not.

The transition rate for a multipole of order L is given by

$$T(L)=\frac{8\pi(L+1)}{L[(2L+1)!!]^2}\omega^{2L+1}B(L) \tag{36}$$

and

$$B(L,J\to J')=\left|\langle JKLK'-K|J'K'\rangle \cdot\int\varphi_{K'}{}'^{*}\mathfrak{M}(L,K'-K)\varphi_K dt\right|^2, \tag{37}$$

where φ_K is the instrinsic state, most conveniently expanded in terms of the $|Nl\Lambda\Sigma\rangle$ basis states. The notation for the vector coupling coefficient is such that

$$|j_3m_3\rangle=\sum_{m_1+m_2=m_3}\langle j_1m_1j_2m_2|j_3m_3\rangle|j_1m_1\rangle|j_2m_2\rangle.$$

The regular transition operators are, neglecting collective terms,[41]

$$EL=\mathfrak{M}_e(L,M)=e(Q+(-1)^LZ/A^L)r^LY_{LM},$$

$$ML=\mathfrak{M}_m(L,M)=\frac{e}{2M}\left(g_\sigma\boldsymbol{\sigma}+g_l\frac{2}{L+1}\mathbf{l}\right)\cdot\nabla(r^LY_{LM}),$$

where $e^2=1/137$, $Q=$ nucleon charge, and the coordinates refer to the coordinate system of the basis states where $|Nl\Lambda\Sigma\rangle=|Nl\rangle Y_{l\Lambda}\chi_{\frac{1}{2}\Sigma}$, $|Nl\rangle$ is the normalized three-dimensional harmonic-oscillator radial wave function, and $\chi_{\frac{1}{2}\Sigma}$ is the Pauli spinor. Thus, to calculate

$T(\tilde{M}1)$ for the 396-keV Lu^{175} transition in Table VIII, we obtain first

$$\tilde{M}1=\frac{eG''}{2M}(2g_\sigma-g_l)(i\boldsymbol{\sigma}\times\mathbf{r})\cdot\boldsymbol{\nabla}(rY_{1M})$$
$$=\frac{eG''}{M}(2.29)(i\boldsymbol{\sigma}\times\mathbf{r})_M\left(\frac{3}{4\pi}\right)^{1/2}.$$

Then we have from Eqs. (36) and (37)

$$T(\tilde{M}1)=\frac{(8\pi)(2)\omega^3e^2G''^2(2.29)(3)}{(1)(3!!)^2\qquad(4\pi)}|\langle\tfrac{9}{2}\tfrac{9}{2}1-1|\tfrac{7}{2}\tfrac{7}{2}\rangle$$
$$\cdot\langle554+|(\boldsymbol{\sigma}\times\mathbf{r})_1|444-\rangle|^2$$

and

$$\langle554+|(\boldsymbol{\sigma}\times\mathbf{r})_1|444-\rangle$$
$$=\langle554+|(\sigma_+z-r_+\sigma_z)|444-\rangle$$
$$=\sqrt{2}\langle554|z|444\rangle$$
$$=\sqrt{2}\langle55|r|44\rangle(4\pi)^{1/2}\int Y_{54}{}^*Y_{10}Y_{44}d\Omega.$$

The integral can be evaluated using[41]

$$\int Y^*_{l'\Lambda'}Y_{LM}Y_{l\Lambda}d\Omega=\left(\frac{(2l+1)(2L+1)}{4\pi(2l'+1)}\right)^{1/2}$$
$$\times\langle l\Lambda LM|l'\Lambda'\rangle\langle l0L0|l'0\rangle$$

and $\langle55|r|44\rangle=(11/2M\omega_0)^{1/2}$ with $\omega_0\sim41A^{1/3}$ MeV gives finally

$$T(\tilde{M}1)=\frac{(16)(2.29)^2}{(15)}\frac{\omega^3e^2\mathfrak{F}^2}{M^3\omega_0R^2}=5.4\times10^{11}\mathfrak{F}^2\ \mathrm{sec}^{-1}.$$

The outstanding example of $M1$ hinderance (3×10^6) is the ground-state decay of the 480-keV level in Ta^{181}. This transition gives $\mathcal{R}_{M1(E1)}=1.4\times10^4\gamma$ where estimating γ at around 0.14 gives $R\sim2\times10^3$. In this example, the $E2$ transition, although itself hindered (see explanation of Table V), constitutes 97% of the decay rate of this state and $\mathcal{R}_{E2(E1)}\sim4\times10^2$ is the more meaningful quantity. Other favorable $M1(E1)$ transitions, identified in Eu^{153}, Dy^{161}, and Np^{237}, are hindered by at most 10^4 although better examples may ultimately be discovered.

To determine the sign of $\mathcal{R}$, we need more information than the approximate wave functions and the decay rate of the regular transition when that transition is strongly hindered. For an $E1(M1)$ transition it is often possible to determine experimentally the relative phase of the $E1$ and $M2$ transition matrix elements. Since the $M2$ transitions are allowed for the $M1$ examples on Table V, the relative $M1$ to $M2$ phase may be computed from the approximate wave functions and thereby the sign of $\mathcal{R}_{E1(M1)}$ may be determined.

It has been pointed out that[60] when the extra-core particle is a neutron and the transition changes the orbital state but not the spin projection ("orbit flip"), the magnetic transition rates are extensively hindered. Table I of Ref. 60 lists several transitions of this type, and these transitions are all allowed for their irregular components. The selection rules will not be found in Table V since the regular transition is also allowed (but retarded due to the zero charge of the neutron) and are given in Table VI. The most favorable example is the 5^mM3 transition in W^{179} ($\frac{1}{2}\frac{1}{2}-[521]\rightarrow\frac{7}{2}\frac{7}{2}-[514]$) which is about 7×10^4 times slower than single-particle estimates. Again using Eq. (31), the asymptotic wave functions give

$$T(\tilde{E}3)=(3.1\ \mathrm{sec}^{-1})\gamma^2\mathfrak{F}^2,$$

or

$$|\mathcal{R}_{M3(E3)}|=38\gamma.$$

For experimental details such as alignment of apparatus, it may be useful to have available transitions which should show no parity nonconserving effects. Electromagnetic transitions among members of the same rotational band are expected to be virtually parity pure, since a given transition matrix element is proportional to its expectation value for the intrinsic state, and the latter must vanish for the irregular transition operators as discussed in Sec. IIIA.

E. Internal Conversion

In the usual approximation[61,62] the transition amplitude for internal conversion, the ejection of a bound atomic electron, is equal to the amplitude for emission of a gamma ray multiplied by a tabulated factor. If the photons are circularly polarized, then the electrons will also be circularly polarized. A significant advantage may be gained by examining the electrons rather than the photons, since the electric and magnetic multipoles do not couple in the same way with bound electrons. This is easily seen for the K shell where a $\frac{1}{2}+$ electron may be ejected as an s wave by an $M1$ transition, but must come out as a p wave for the $E1$ transition. The difference in coupling becomes pronounced at high Z for low-energy transitions where the overlap between tightly bound initial-state electrons and the slow final-state continuum electron is very sensitive to the angular momentum of the latter. For special values of Z, E, and L, the ratio of the magnetic-multipole to electric-multipole K-conversion coefficients, β_L/α_L in the notation of Rose, may be as large as 10^3 or greater. The degree of electron polarization will then be of the order $(\beta_L/\alpha_L)^{1/2}$ times the EL photon polarization. The actual expression is somewhat more complicated due to the contribution of two partial waves in general to the

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[60] H. Morinaga and K. Takahashi, Nucl. Phys. **38**, 186 (1962).
[61] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).
[62] M. E. Rose, *Multipole Fields* (John Wiley & Sons, Inc., New York, 1955), p. 65.

internal conversion process. For example, in the K shell, and $M1$ multipole couples to both $s_{1/2}$ and $d_{3/2}$ continuum states while the $E1$ couples to $p_{1/2}$ and $p_{3/2}$. The final polarization is a weighted average of the interference between the angular momentum $\frac{1}{2}$ and $\frac{3}{2}$ waves which precludes simply writing the result in terms of $(\beta/\alpha)^{1/2}$ although this should be of the correct order of magnitude. The properly weighted result for electron polarization in $EL(ML)$ conversion from the K shell can be written $f_L(\beta_L/\alpha_L)^{1/2}\mathcal{RF}$, where

$$f_L=\frac{2[(L+1)(R_3+R_4)-(2L+1)R_5+R_6]}{[L|R_3+R_4+2R_6|^2+(L+1)|R_3+R_4-[2+(1/L)]R_5-(1/L)R_6|^2]^{1/2}}\cdot\frac{(R_1+R_2)}{[(2L+1)|R_1+R_2|^2]^{1/2}},$$

and $|f_L|$ varies between $(4/5)^{1/2}$ and $(8/5)^{1/2}$ if R_3 to R_6 have the same sign. The R_i are defined in Ref. 62. Tables of the individual R_i do not seem to be available; however, taking $|f_L|\approx 1$ is probably not too bad for a rough approximation. If the transition is $ML(EL)$ the ratio β/α is approximately inverted provided the L_{II} line is examined instead of the K or L_I lines, although resolving L_{II} from the L lines may be experimentally more difficult than resolving the K from the L lines.

F. Scattering Experiments

In most scattering processes the electromagnetic and/or nuclear forces will dominate and virtually obliterate any sign of the parity nonconserving interactions. The transmission of thermal neutrons through a crystal constitutes an exceptional situation, since the Coulomb force is absent and the nuclear scatterings add coherently to appear simply as an "index of refraction" for neutron waves in the crystal. Magnetic and spin-dependent nuclear interactions can be circumvented in crystals composed of atoms without static magnetic moments and nuclei having zero spin or negligible spin-dependent interaction. Thus, a thermal neutron may propagate through a crystal essentially as a free particle, and under these conditions the parity nonconserving forces may produce observable effects.

A plane wave of momentum k, in traversing a length h of matter with refractive index n, will acquire a phase factor $\exp(i\varphi)$ where

$$\varphi=(n-1)kh. \tag{38}$$

The index of refraction for neutrons is given to an excellent approximation[63] by $n=1-2\pi Nf/k^2$, where N is the number of nuclei per unit volume and f is the scattering length due to the nuclear forces. The parity nonconserving interaction gives a contribution to the index of refraction dependent on the sense of spin polarization along the momentum direction. The total scattering length is then $f+f'$ for neutrons polarized parallel to k, $f-f'$ for antiparallel, where f', derived in Appendix B, is

$$f'=2G'MR^3kCZ/3A. \tag{B5}$$

The quantity C is a correction for the distortion of the neutron amplitude inside the nucleus, given approximately (see Appendix B) by

$$C\approx 3f/2R^2(f-R), \tag{B4}$$

if f and the nuclear radius R are measured in fermis (10^{-13} cm).

Consider an incident neutron polarized in the positive x direction with momentum k in the positive z direction: the polarization is then (1,1) if described in terms of spin amplitudes for spin parallel to k, (1,0), and antiparallel, (0,1). The spin amplitude after a distance h in the matter will be $(e^{-i\varphi},e^{+i\varphi})$ multiplied by an unimportant mutual phase factor due to the nuclear forces, i.e., the scattering length f. The expression $(e^{-i\varphi},e^{+i\varphi})$ corresponds to a polarization with components $\cos 2\varphi$ along the positive x direction and $\sin 2\varphi$ along the positive y direction, thus the neutron polarization has been rotated by an angle 2φ in the sense of a right-hand screw (if φ is positive) as illustrated in Fig. 4. The angle 2φ is given by Eqs. (38) and (B5) to be

$$2\varphi=-(2G'/\rho_0)\rho hCZ/A,$$

where $(2G'/\rho_0)=9.0\times10^{-9}$ rad cm^2 g^{-1}, and ρ is the density of the crystal. For example, bismuth, adopting the values $f=8.63F$, $R=7.13F$, $\rho=9.80$ g/cm^3, $h=10^3$ cm, $Z=83$, and $A=209$, gives the rotation $2\varphi=-5.8\times10^{-6}$ rad, where C comes out to be $+0.17$.

Although a rotation of only 5.8×10^{-6} rad after passing through 10 m of bismuth is an exceeding slight effect, it may not be completely out of the question to detect such a small rotation. Consider an experiment in which a beam of neutrons polarized in the positive x direction are incident on the "rotator," as in Fig. 4, and a polarization analyzer is aligned to detect neutrons polarized along the y direction. The ratio, r, of the counting rate for $+y$ polarization to $-y$ polarization will be $r=(\frac{1}{2}+2\varphi)/(\frac{1}{2}-2\varphi)\simeq 1+8\varphi$. Very intense

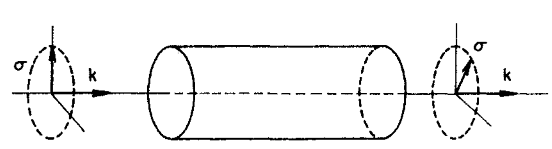

FIG. 4. Rotation of the neutron polarization for $\varphi>0$.

[63] D. J. Hughes, *Neutron Optics* (Interscience Publishers, Inc., New York, 1954), p. 24.

beams of polarized neutrons can be produced and it is not necessary that the polarization be 100% for this experiment. Furthermore, the alignment need not be exact since all one really wishes to measure is r(rotator in)$-r$(rotator out)$=8\varphi$. The polarization will precess in any stray magnetic fields by an amount $0.079\int_0^h H_z dz$ rad cm^{-1} G^{-1}, and to keep the net precession small, one must limit the average value of H_z to less than 2.5×10^{-3} G. The precession would cancel in the difference r(rotator in)$-r$(rotator out) were it not for the diamagnetism of the rotator. Consequently, the net precession must be small compared to $\varphi/4\pi\chi$, where χ is the magnetic susceptibility of the rotator, and for the above example the average value of H_z must then be less than 5×10^{-3} G.

It is curious that the angular change resulting from such an "optical rotation" [similar to the rotation of plane polarized optical radiation in a solution of right (or left) handed molecules such as sugar] of neutrons is of the same order of magnitude as the other effects discussed in this paper. Here, the interaction energy is reduced by a factor of $\rho/M\rho_0\sim10^{-15}$ while the interaction length is increased by a factor of about $h/R\sim10^{15}$, and the two factors roughly cancel.

V. SUMMARY

In this paper we have adopted the current-current hypothesis for the weak interactions and from this deduced the form of the parity nonconserving internucleon potential. This potential is reduced to an approximate single-particle interaction and it is observed that such an approximate interaction can be removed from simple phenomenological Hamiltonians by a gauge transformation. On the basis of this simplifying approximation, a search is made for experiments that might reveal characteristic effects from a parity nonconserving interaction.

The experiments so far performed do not seem to have been sensitive enough to test the presence of the self-self weak interaction; however, these experiments have served the important function of showing the amount of parity nonconserving interaction in the nuclear forces to be extremely small compared to the strength of the nuclear parity conserving forces. Of the experiments discussed, beta decay appears to be the least sensitive. Observation of the "optical rotation" of neutrons passing through matter seems remote, although it is amusing to find a quantum mechanical effect as subtle as parity nonconservation displayed in an (almost) classical experiment. The parity-forbidden alpha decay and the various experiments involving gamma decay both fall tantalizingly close to existing experimental capabilities. The alpha-decay experiments cannot give, however, the sign of G. A test that may well be feasible is to look for a circular polarization of the internal conversion electrons from Lu^{175} which should be polarized by about $(\beta_1/\alpha_1)^{1/2}\mathfrak{RF}\approx0.004$. Better examples may come to light or the techniques may be refined to detect such small polarizations.

We may have overlooked the really sensitive tests, and this paper is offered with the hope of stimulating new ideas in this direction.

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APPENDIX A: DERIVATION OF THE SINGLE-PARTICLE APPROXIMATION INTERACTION

The internucleon parity nonconserving interaction (14) is reduced to a single-particle interaction by averaging one of the nucleons over the possible states of a symmetric core. Consider a nucleus with one particle in addition to filled shells. The initial state of the particle is denoted by ν, the final state by u, and the filled-shell states by w, with such subscripts to be added as necessary if certain of the quantum numbers are to be exhibited explicitly. The many-particle matrix element reduces[64] to the two-particle matrix element given below in Eq. (A1).

$$\langle f|V|i\rangle=\sum_k\int d(1)d(2)u^*(1)w_k^*(2)V(1,2)\times\{\nu(1)w_k(2)-\nu(2)w_k(1)\}. \quad \text{(A1)}$$

Only the core neutrons (protons) will interact with an extra-core proton (neutron) as can be seen from the isotopic spin dependence of $V(1,2)$ given in Eq. (16). Only the exchange term in the brackets of (A1) then contributes giving

$$\langle f|V|i\rangle=-\sum_k\int d(1)d(2)u^*(1)w_k^*(2)V(1,2)\nu(2)w_k(1),$$

where the sum over k is now understood to include only the appropriate nucleons. The core is assumed to have zero total angular momentum and we then write

$$\sum_k w_k=\sum_m^{2l+1} w_{lm}\sum_\sigma \chi_\sigma, \quad \text{(A2)}$$

where χ_σ is the nucleon spin state. This expression is valid for a complete shell in LS coupling where the shell contains $2(2L+1)$ nucleons. For a filled shell in

[64] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, New York, 1951), p. 173.

jj coupling ($2j+1$ nucleons), we have

$$\sum_k w_k = \sum_m^{2j+1} w_{jm} = \sum_m^{2l+1} \sum_\sigma^2 C_m{}^\sigma w_{lm} \chi_\sigma$$

with

$$C_m{}^\sigma = C_{-m}{}^{-\sigma}, \quad \sum_\sigma^2 \sum_m^{2l+1} (C_m{}^\sigma)^2 = 2j+1,$$

and since all projections m (from $+j$ to $-j$) are summed over, we have

$$\sum_m^{2l+1} C_m{}^\sigma w_{jm} = \sum_m^{2l+1} C_m{}^{-\sigma} w_{jm} = \sum_m^{2l+1} C_m w_{jm},$$

which reproduces (A2) with a weight factor. Using (A2), the form of Eq. (14) for the interaction, and treating separately specific spin amplitudes of u and ν then allows $\langle f|V|i\rangle$ to be factored into separate spatial and spin parts. We want to find $V_{\text{effective}}$ that acts between u and ν to give the same result as $\langle f|V|i\rangle$, as opposed to evaluating $\langle f|V|i\rangle$ which would require a specification of the states u and ν. The spin operator $(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)$ gives

$$\sum_\sigma \chi_u(1)\chi_\sigma(2)(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)\chi_\nu(2)\chi_\sigma(1).$$

From rotational invariance it is sufficient to consider $(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)_z$ which gives $\sum_\sigma (\nu-\sigma)\delta_{\nu\sigma}\delta_{u\sigma}=0$ and consequently the first term in Eq. (14) averages to zero. The identity $i\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_1=(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)P^\sigma$, where

$$P^\sigma = \tfrac{1}{2}(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

is the spin-exchange operator, gives for the second term in Eq. (14)

$$-\chi_u(1)\{\sum_\sigma \chi_\sigma(2)(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1)\chi_\sigma(2)\}\chi_\nu(1) = +2\chi_u(1)\boldsymbol{\sigma}_1\chi_\nu(1).$$

The spatial factor from this term is [defining $\mathbf{p}_{21}=(\mathbf{p}_2-\mathbf{p}_1)$]

$$\int d(1)d(2)u^*(1)\sum_m w_m{}^*(2)[\mathbf{p}_{21},f(1,2)]_- \times\nu(2)w_m(1). \quad \text{(A3)}$$

In the limit $f(1,2)\to\delta(1,2)$, we can employ a second identity

$$[\mathbf{p}_{21},\delta(1,2)]_- = \{\mathbf{p}_{21},\delta(1,2)\}_+ P^q, \quad \text{(A4)}$$

where P^q is the coordinate exchange operator. The proof rests on the observation that integrals over $d(1)d(2)$ of $\delta(1,2)$ multiplied by nonsingular antisymmetric functions of 1 and 2 are identically zero. The operators in (A4) can connect only states of opposite symmetry since $\mathbf{p}_{21}$ operating on a symmetric state gives an antisymmetric state and vice versa. Hence, only one of $\mathbf{p}_{21}\delta(1,2)$ or $\delta(1,2)\mathbf{p}_{21}$ has a nonvanishing contribution, depending on whether the initial or final state is antisymmetric, respectively, and so $[\mathbf{p}_{21},\delta(1,2)]_-$ and $\{\mathbf{p}_{21},\delta(1,2)\}_+$ are equal within a sign, this being provided by P^q. The approximate validity of (A4) for a force with finite range then depends on the range a being short compared to the wavelength, or more quantitatively $3k^2a^2/5\ll 1$. This criterion is satisfied, although not very strongly, for nuclei where taking $a\sim M_V{}^{-1}$ and assuming an average nucleon kinetic energy of about 30 MeV gives $3k^2a^2/5$ equal to 0.10. Combining (A3) and (A4) gives

$$\int d(1)u(1)[\sum_m \int d(2)w_m{}^*(2)\{\mathbf{p}_{21},f(1,2)\}_+ w_m(2)]\nu(1).$$

The symmetry of the substates averages $\mathbf{p}_2$ to zero giving finally for the effective interaction

$$V_{\text{eff}} = \frac{\sqrt{8}\lambda G(\mu^v+1)}{8M}\boldsymbol{\sigma}_1\cdot\{\mathbf{p}_1,\rho(1)\}_+, \quad \text{(A5)}$$

where $T_{12}=\frac{1}{2}(1-\tau_z{}^1\tau_z{}^2)P^\tau$ has been used to obtain

$$\rho(1)=\sum_m \int d(2)w_m{}^*(2)f(1,2)\tfrac{1}{2}(1-\tau_z{}^1\tau_z{}^2)w_m(2)$$

$$\approx\sum_m \int d(2)w_m{}^*(2)f(1,2)w_m(2)\cdot\tfrac{1}{2}[1+\tau_z{}^1(N-Z)/A].$$

The quantity $\rho(1)$ is then the effective nuclear isotopic spin density, weighted for the range of the interaction $f(1,2)$, seen at the position of nucleon 1. Note that the sum is over *all* nuclear states in the core. Definition of $\rho(1)$ in (A5) together with the arguments of Sec. IID then give

$$\lambda\rho(1)\approx\lambda'\rho_{\text{nucl}}(1)\approx\lambda'\rho_0\tfrac{1}{2}[1+\tau_z{}^1(N-Z)/A]$$

and (A5) becomes finally

$$V_{\text{eff}} = \frac{(8)^{1/2}\lambda' G(\mu^v+1)\rho_0}{8M}\boldsymbol{\sigma}\cdot\mathbf{p}(1+\tau_z(N-Z)/A)$$
$$= G'\boldsymbol{\sigma}\cdot\mathbf{p}(1+\tau_z(N-Z)/A) = G''\boldsymbol{\sigma}\cdot\mathbf{p}, \quad \text{(A6)}$$

where $G'=0.22\times 10^{-7}$.

Although the shell model has been used at a few points in this section, the only assumption actually required is that the core remain unchanged and have zero spin. If Eq. (14) is modified to have isotopic spin dependence

$$\tau_+{}^1\tau_-{}^2+\tau_+{}^2\tau_-{}^1+\epsilon_1(1+\tau_z{}^1\tau_z{}^2)+\epsilon_2(1-\tau_z{}^1\tau_z{}^2), \quad (17)$$

then the isotopic spin dependence of (A6) is modified to

$$[a+b\tau_z(N-Z)/A],$$

where

$$(\mu^v+1)a=(\mu^v+1)+2\epsilon_2,$$
$$(\mu^v+1)b=(\mu^v+1)(1-2\epsilon_1)+2(\epsilon_2-\epsilon_1).$$

The contribution of Eq. (14) to the electromagnetic coupling (14′) averages to zero as can be seen in the reduction from (A1) to (A3).

APPENDIX B: IRREGULAR SCATTERING LENGTH FOR LOW-ENERGY NEUTRONS

That part of the scattering length for low-energy neutrons sensitive to longitudinal polarization of the neutrons is computed for a simple nuclear model. Let $\psi_k^\dagger(1)$ represent the wave function of the neutron propagating with momentum k in the positive z direction where

$$\psi_k^\dagger(1)=\sum_{l=0}^{\infty} i^l(2l+1)e^{i\delta_l}P_l(x)R_{kl}(r) \quad x=\hat{k}\cdot\hat{r},$$

assuming a spherically symmetric nuclear potential. If interaction with the nuclear potential were ignored, then $R_{kl}(r)\to j_l(kr)$, where $j_l(x)$ equals $(\pi/2x)^{1/2}\times J_{l+1/2}(x)$, $\delta_l=\delta_0$, and the wave function becomes $\exp(ikz+i\delta_0)$. However, this wave function is assumed known, and the correction due to the parity nonconserving interaction of (A6) is

$$\frac{M}{2\pi}\int\frac{e^{ikr_{12}}}{r_{12}}(-iG''\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\psi_k^\dagger(2)d(2)\,, \qquad \text{(B1)}$$

where $r_{12}=|\mathbf{r}_1-\mathbf{r}_2|$ and the integral extends over the nuclear volume. From symmetry ∇_x and ∇_y give no contribution, and taking the incident neutron to be polarized in the positive z direction, (B1) becomes

$$\frac{-iMG''}{2\pi}\int\frac{e^{ikr_{12}}}{r_{12}}\nabla_z(2)\psi_k^\dagger(2)d(2)\,. \qquad \text{(B2)}$$

The coefficient of $\exp(ikr_1)/r_1$ in the limit $r_1\to\infty$ of (B2) is the scattering length f' and for $k\to 0$, (B2) together with $\nabla_z=x(\partial/\partial r)+[(1-x^2)/r]\partial/\partial x$ gives

$$f'=-iG''M\sum_{l=0}^{\infty} i^l(2l+1)e^{i\delta_l}\int_0^R\int_{-1}^{+1} r^2drdx \times\left\{xP_l(x)\left[R_{kl}'(r)+\frac{l+1}{r}R_{kl}(r)\right]-\frac{l+1}{r}R_{kl}P_{l+1}(x)\right\}.$$

This expression is readily simplified using $\int_{-1}^{+1}P_lP_{l'}dx=(2/2l+1)\delta_{ll'}$ and $P_0(x)=1$, $P_1(x)=x$ to give

$$f'=2G''Me^{i\delta_1}\int_0^R\left(R_{k1}'+\frac{2}{r}R_{k1}\right)r^2dr\,. \qquad \text{(B3)}$$

For a square-well potential $R_{kl}=[k/Kj_0(KR)]j_1(Kr)$ for $r\leq R$, where $K=(2MU)^{1/2}$ with U the depth of the potential. The coefficient of j_1 correctly matches the interior solution to the incident (asymptotic) plane wave provided the phase shift $\delta_1\approx 0$. For this potential, $\tan\delta_1=(kR)^3[C(\zeta)-1]/3$ where $\zeta=KR$ and

$$C(\zeta)=3j_1(\zeta)/\zeta j_0(\zeta)=3(1-\zeta\cot\zeta)/\zeta^2\,. \qquad \text{(B4)}$$

Expression (B3) gives

$$f'=2G''MkR^3C(\zeta)/3\,. \qquad \text{(B5)}$$

In the limit $U\to 0$ we have $K\to k\approx 0$ and $C(0)=1$ giving the same final result for f' as would be obtained by initially assuming $\exp(ikz)$ for the unperturbed wave function, i.e., neglecting the nuclear forces. The correction for these forces is then the expression given in (B4) if the approximations discussed above are employed. For thermal neutrons scattered by nuclei, the phase shift δ_1 will become important only if $C\gtrsim 10^{15}$. The expression (B4) is rather inconvenient for estimating C, since ζ for heavy nuclei may be 10 or larger. Small relative uncertainties in ζ will then make large uncertainties in $\cot\zeta$. We can make use of the experimentally determined quantities f and R by way of the relation $R/(R-f)=\zeta\cot\zeta$ given from the square-well model. The correction C is then given from (B4) to be

$$C=3f/2R^2(f-R)\cdot(1/MU)\approx 3f/2R^2(f-R)\,, \qquad \text{(B6)}$$

if f, R are in fermis since $MU\sim 1F^{-2}$ if U is estimated to be about 40 MeV. Expression (B6) also is probably less sensitive to the specific model for the nuclear interaction.